Measurement of Information Superiority Based on OSPA Distance

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Keywords: Information superiority, OSPA distance, Measurement, Agent.

Abstract. Information superiority is an important concept in Network Centric Operation (NCO), but the research on its measurement is not based on rigorous mathematical foundation. That causes the problems such as lack of operability, insufficient in explanation, limited scope of application and so on. This paper provides a new method of measuring information superiority based on optimal sub-pattern assignment (OSPA) distance, and redefines information superiority by calculating the OSPA distance between different information sets. The new model is preliminarily verified with the simulation of homogeneous coordination based on Agent and the results show that it is operable, flexible and applicable.

Introduction

In Joint Vision 2020, Information superiority is defined as the capability to collect, process, and disseminate an uninterrupted flow of information while exploiting or denying an adversary’s ability to do the same\textsuperscript{[1]}. Describing and measuring information superiority is a very important issue in many aspects of C\textsuperscript{4}ISR system construction, such as requirement analysis, efficiency evaluation, operational application and so on. And it has very important significance for understanding the information operations, enhancing situational awareness and improving cooperation efficiency\textsuperscript{[2]}.

Recent research on information superiority measurement mainly focus on information quality, and establishes the measurement model by the index synthetic method. Alberts\textsuperscript{[3]} proposed the concepts of information superiority completeness, accuracy and timeliness. After that, followed studies were carried out to quantify these concepts by index synthetic method\textsuperscript{[4-7]}. However, there are several problems in these studies. Firstly, in index synthetic method, only one index can not reflect the real tend of information superiority. Paradoxically, synthesized function has to choose the weight of index, which is very difficult to determine accurately. That causes a series of problems in practicability and operations. Secondly, since the index synthetic method is originally designed for efficiency measurement of complete battle system, local or dynamic characteristics of information superiority is difficult to describe. Thirdly, lack of strict mathematical basis, the index synthetic method is not easy to be estimated by communication and network parameters, and also it is not easy to extend.

To solve these problems, this paper tries to establish a new information superiority model by introducing the optimal sub-pattern assignment (OSPA) distance. OSPA distance, originally designed for multi-target filtering, is usually used for comparing the difference between two given sets, one of which to be compared is a real value set. Based on strict mathematical definition, it can reflect the feature of information quality directly, avoiding the selection of the index weight. Also, it is sensitive with the number of set elements, which gives a more stable performance when the element number changes fast. All of these characteristics make it suitable for information superiority measurement.

The structure of the paper is as follows. In the first section, the definition of OSPA distance is introduced. Next, information quality and superiority model is proposed by calculating OSPA distance of sets. At last, by a homogeneous coordination model based on Agent, the new model is preliminarily simulated and verified.
**Brief Introduction of OSPA Distance**

In set theory, the pioneering work of distance metric between two sets is Hausdorff distance[8], which measures a maximum mismatched degree of two sets. But it is not sensitive to element number. Hoffman and Mahler partially solved this problem. They defined the multi-objective mismatch distance[9] from Wasserstein distance[10]. Schumacher and his cooperators pointed the shortages of Hoffman-Mahler distance, and they proposed a new method of consistency distance for multi-objective filtering[11], namely OSPA distance. OSPA distance, in which both the number and the state of set elements are considered, is the latest progress in the field of distance metrics between sets[12]. The definition of OSPA distance is as fellows.

Let $W$ be a finite set. Let $X = \{x_1, ..., x_m\}$, $Y = \{y_1, ..., y_n\}$ are two finite subsets of $W$. Without loss of generality, assume that $m \leq n$. If $x, y \in W$ and $d(x, y)$ is a basic distance in $W$, then $d(x, y) := \min(c, d(x, y))$. Let $\Pi_k$ be the set of all permutations of $\{1, 2, 3, ..., k\}$, $k \in \{1, 2, ..., n\}$. For any given number $c$ and $p$, if $1 \leq p < \infty$ and $c > 0$, then define the OSPA distance between $X$ and $Y$ as

$$
\overline{d}_p(c)(X, Y) := \left( \frac{1}{n} \min_{\pi \in \Pi_m} \sum_{j=1}^{m} d(c, (x_j, y_{\pi(j)}))^p + c^p(n-m) \right)^\frac{1}{p} \tag{1}
$$

After summation operator of OSPA distance equation, there are two terms of different meanings. The first term represents the sum of distance between matched elements, and the second term represents the distance of isolated elements. It should be noticed that if the distance between the two matched elements is larger than $c$, then the contribution to $\overline{d}_p(c)(X, Y)$ is $c$, just like isolated elements. That means the number $c$ can be regard as punishment cost of isolated elements.

**Information Superiority Measurement Model Based on OSPA Distance**

In this section, we focus on information superiority measurement model based OSPA distance. Firstly, some importance concepts such as battlefield information space are defined. Then the OSPA distance between situation awareness information set and objective information set is calculated and the new information superiority measurement model is given. Lastly, a discussion shows the advantages of the new model.

**Battlefield Information Space**

Let the number of the targets in battlefield be $N$ and the number of one's own combat unit be $M$. Define that the collection of all information generated by $N$ targets and $M$ combat units is an information set, namely *Battlefield Information Space*. Let there be $Z$ kinds of information components in the space $W$. Generally, $Z$ is a relatively large number, which leads design of algorithm on $W$ is difficult. However, by selecting the information we care about, an information subset $\tilde{W}$ with less dimensions can be designed to solve this problem.

Let battlefield situation awareness information set of one side be $U$ and battlefield objective information set is $V$, where $U, V \subset \tilde{W}$. Let $T_1$ and $T_2$ be the beginning time and end time. If $t \in [T_1, T_2]$ represents present time, then at time $t$, the situation awareness information set of $i$ th unit is $\hat{X}_i(t)$, and battlefield objective information set is $X(t)$, where $\hat{X}_i(t) \subseteq U, X(t) \subseteq V, i = 1, 2, ..., N$. $\hat{X}_i(t)$ reflects the information one side obtains at $t$, and the difference compared to $X(t)$ reflects the ability to get and share information in battlefield. Let $F$ be the distance function on space $\tilde{W}$, namely $F: \tilde{W} \times \tilde{W} \to R$. Define the distance of $i$ th unit between situation awareness and objective information at time $t$ as $D_i(t) = F(\hat{X}_i(t), X(t))$. If $D_i(t)$ can be calculated with rigorous
mathematical methods, the information superiority model will be established by comparing \( D_i(t) \) of two sides. Fortunately, the problem can be solved by OSPA distance.

**OSPA Distance Between Situation Awareness Information Set and Objective Information Set**

Let \( \hat{\mathbf{x}}_{i,j}(t) \in \hat{\mathbf{X}}_i(t) \) be the situation awareness information vector describing \( j \)th target and got by \( i \)th unit. In the same way, Let \( \mathbf{x}_j(t) \in \mathbf{X}(t) \) be the objective information vector describing \( j \)th target, then \( D_i(t) \) can be written as

\[
D_i(t) = \left( \frac{1}{N + M} \sum_{j=1}^{m} \min \left( d^{(c)}(\hat{x}_{i,j}, x_{\pi(j)})^p + c^p (N + M - m) \right) \right)^{\frac{1}{p}}
\]

where, \( d^{(c)}(x, \hat{x}) = \min(c, |x - \hat{x}|) \).

As mentioned in the first section, the OSPA distance equation includes two terms. In equation (2), the first term is the distance of matched elements, representing the influence of known information. The second term is the distance of isolated elements, representing the influence of unknown information. The parameter \( c \) can be understood as the punishment cost fee of unknown information, because the influence of known information is equal to unknown information when the distance between two elements is larger than \( c \). The parameter \( p \) reflects the weight of matched and isolated elements in total distance. The larger the value of \( p \), the weight of isolated elements is bigger, which means unknown information plays a relatively more important role than known information. Therefore, \( p \) can be understood as the adjustment factor of known and unknown information influence in total distance. Because of the parameters \( c \) and \( p \), it is more controllable and adjustable to establish information superiority model based on OSPA distance.

The equation (2) shows the distance between two information sets on single unit and target. For describing the features of the whole system during a period of time, it is necessary to take an average. So the system average distance between situation awareness and objective information is

\[
D = \frac{1}{M(T_2 - T_1)} \sum_{i=1}^{T_2} \sum_{j=1}^{M} \left( \frac{1}{N + M} \sum_{j=1}^{m} d^{(c)}(\hat{x}_{i,j}(t), x_{\pi(j)}(t))^p + c^p (N + M - m) \right)^{\frac{1}{p}}
\]

(3)

**Information Quality and Information Superiority Model**

On the basis of the above chapter, Information Quality \( Q \) can be defined as \( Q = 1 - D \). Information quality represents the similarity degree compared to objective information set and reflects the absolute value to obtain information.

Let red represent one side and blue the other, then information quality of two sides can be expressed by \( Q_R \) and \( Q_B \). Define Information Superiority \( S_{R,B} \) as \( S_{R,B} = Q_R - Q_B \). Information superiority represents the information quality disparity between two sides, which is a relative concept and reflects the ability gap of the two sides to obtain information. If \( S_{R,B} \) is larger than a certain threshold \( S_{th} \), it can be understood that the red side basically controls the battlefield information space, which means the information of the red side is far more close to objective information set than the blue side.

**Discussion**

According to the definition of information completeness and information accuracy[7], it is easy to know that the first term after summation operator in equation (3) is related to the information accuracy, and the second term is related to the information completeness. In some conditions, it can degenerate into information accuracy or completeness. For example, if \( c = \infty, p = 1 \), then \( Q = \frac{m}{n} \), namely
On the other hand, OSPA distance, based on strict mathematical foundation, is satisfied with qualitative, symmetry and triangle inequality. Thus, the convenient and efficient algorithms for common distance can be directly applied to it. Besides, the new model can be estimated by network, communication and other parameters, also can be used for estimating the combat effectiveness.

Another advantage is that $W$ can be selected flexibly, which means the new method can describe the local characteristics of information superiority and the algorithm complexity on $W$ can be controlled. That is very important in some application scenarios, such as the inhomogeneous distribution or too large dimension of information.

**Simulation and Verification**

In this section, a homogeneous coordination model based on Agent is used to simulate and verify the new information quality model.

**Homogeneous Coordination Model Based on Agent**

Let $M$ red unit and $M$ blue unit is randomly put into the $B \times B$ grid. Each unit has same abilities, including movement, sense, communication, information fusion, decision and strike. The time is discrete and every unit acts as the order of observation, orientation, decision and action (OODA Loop) at a unit time. Only one of the two actions, movement and strike, is allowed to execute once. There is no error in the information directly sensed from the unit itself, but there are errors in the information obtained from other units by communication, because it is only allowed to use after a unit time.

**Preliminary Simulation Results**

Let $c = 4$ and $p = 1$. The unit number of each side is 50 and the sense range of two sides is 3. The Communication range of red changes from 1 to 20 and there is no communication in blue sid

![Graph](image-url)

Figure 1. Performance curve of information quality.

Figure 1 shows the information quality changes with time and communication range. The left part shows $Q_B$ remain the same and $Q_R$ increase rapidly with time, then decreased gradually. That proves communication brings positive impact on information quality. after a period of time, the gain from communication tends to saturation and the decline of the total information amount caused by the death of combat unit leads to a decrease in the quality of information. In the right part, $Q_R$ increases rapidly at first, then the growth rate slowed to stable. It can be understood that the communication brings the positive gain on information quality, and with the expanding of communication range, the
gain of communication become saturated. That means the efficiency of resource input and output changes under different conditions. The results show the method based on OSPA distance can reflect the local dynamic characteristics.

Some meaningful conclusions will be got by focusing on the comparison between different methods in the right part. When the communication range increases, the unit-average information accuracy reduces and system information completeness almost remains constant. Obviously, they don't increase with information quality as expectation and neither of them can completely reflect the real characteristics of information quality. The unit-average information accuracy decreases but the information quality increases, indicating that information with errors still has a positive gain on the latter. Although the unit-average information completeness, which can be considered as the degenerate form of OSPA information quality when $c = \infty$ and $p = 1$, has the same growth trend as the information quality, it grows significantly faster. In spite of partly describing the real feature, it does not reflect the loss brought by the decline of information accuracy. Therefore, it is difficult to reflect the real characteristics of information quality by single index in the index synthetic method.

**Discussion**

The results prove that the new information superiority measurement method based on OSPA distance is operable and can be achieved through the simulation based on Agent. In some conditions, it can be degraded to the traditional method. Without index synthetic, only index of traditional method is not enough to accurately reflect the real trend of information superiority. On the contrary, the new OSPA method can accurately describe information superiority and there is no need to synthesize the indexes, which makes it more practical. Also, it can well reflect the local dynamic characteristics of information superiority. Besides, because of the rigorous mathematical foundation, the new model can be estimated by a certain mathematical method.

**Conclusion**

This paper proposes a new method of information quality measurement by using OSPA distance to quantify the difference between information sets, and preliminary results show that the new method is operable, compatible and flexible. After further improvement, the new method can be widely used in various fields of battlefield situation assessment and C4ISR system construction.

**References**


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