Establishment and Application of Overbooking Model of Single Flight Departure Time Distribution

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Abstract. In order to put forward theoretical foundation for the airlines to strengthen the airline overbooking decision-making systems and do the best to protect the interests of both airlines and passengers. Apply mathematical statistics and probability theory. Analyzed the methods in accordance with the time line, and explains the problems of overbooking, and takes Nanjing Lukou International Airport as an example to carry out the collection and collation of single flight basic data. Based on the model hypothesis is defined. Through the MATLAB software, which verifies the correctness and effectiveness of the model. Research conclusions, according to laws operational and risk aversion mechanism, the making decision of operation method of airline passenger overbooking behavior is scientificity and feasibility.

Introduction

In recent years, the global passenger air transportation industry has experienced a “blowout” growth. In fact, the contradiction between the supply and demand of passenger transportation in the aviation market is very prominent, for passengers, the situation of Denied Boarding often happens during boarding due to the influence of overbooking and profitability of flights. At this time, conflicts between passengers and airlines easily occur. It can be seen that it is important to objectively and scientifically judge the oversold operation mode of flight departure time distribution. The scientific and rational application of flight oversold strategies will help improve the ability of revenue management of airline ticket oversold.

Literature Review

A large number of scholars have researched around the quantitative model and sales strategy of air tickets. Kong Xiangfen (2016) verified the applicability of the two models through civil aviation service data based on the grayness theory of the GM (1,1) model and the Markov chain steady state theory[1]. Li Wenting (2017) believed that civil aviation services are an important part of transportation services, and analyzed the management ideas of civil aviation services in China [2]. Xie Wenlu (2018) traced the legal source of flight overbooking according to the frequent occurrence of flights overbooking [3]. Zhang Nan (2018) analyzed the current market situation of China and the countries along the “Belt and Road” and looked forward to the future market potential [4]. Wang Peng (2019) researched from the economic attributes of aviation products and predicted the future development trend of China's aviation supply market [5].

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Problem Description

As far as airlines are concerned, the ultimate purpose of the oversold operation method is to ensure that the number of passengers who are denied boarding and vacancies during the check-in process are minimized when the flight departs, and that the outbound flight is sold as early as possible to maximize the benefits by reducing the cost of flight seats. In order to find the equilibrium point
between the total aviation revenue and the maximum number of seats available for sale, it is necessary to consider the equilibrium relationship between the seat cost (SC) and denied boarding cost (DBC). Therefore, in order to scientifically and reasonably seek the equilibrium point of revenue management, this article attempts to establish an oversold model of flight departure time distribution.

**Collection and Arrangement of Basic Data**

In order to analyze the flight departure time distribution model, taking Nanjing Lukou Airport as an example, the initial data of the departure time of Nanjing-Chongqing, Nanjing-Xiamen, Nanjing-Beijing, Nanjing-Kunming during approximately one week in April 2017 was collected. Due to space limitations, the article lists the initial data of Nanjing-Xiamen as shown in Table 1.

<table>
<thead>
<tr>
<th>Time</th>
<th>7</th>
<th>10</th>
<th>11</th>
<th>13</th>
<th>17</th>
<th>18</th>
<th>19</th>
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<td>1</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
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<tr>
<td>4.2</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4.3</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4.4</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
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<tr>
<td>4.5</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
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<td>4.6</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4.7</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>27</td>
<td>6</td>
<td>40</td>
<td>26</td>
<td>12</td>
<td>9</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

**Model Calculation of Flight Departure Time by Period**

By comparing different fitting methods and analyzing the fitting error situation according to statistical methods, we can know that Gaussian distribution or spline fitting are two better fitting methods, and the fitting error of Gaussian distribution is relatively small. So it is planned to use Gaussian distribution method to mathematically approximate the flight departure time distribution. The flight departure curve of the morning, noon, afternoon, and evening time periods is as follows. Time curve of flight departure volume in the morning time. See Figure 2, Figure 3 and Table 4, Table 5. The same methods, time curve of flight departure volume at noon time, in the afternoon time and during night time.
Table 4. Fitting of errors.

<table>
<thead>
<tr>
<th>Evaluation index</th>
<th>General model Gauss1</th>
<th>Smoothing spline</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE</td>
<td>352.1</td>
<td>1890</td>
</tr>
<tr>
<td>R-square</td>
<td>0.9917</td>
<td>0.9552</td>
</tr>
<tr>
<td>Adjusted R-square</td>
<td>0.975</td>
<td>0.8507</td>
</tr>
<tr>
<td>RMSE</td>
<td>18.76</td>
<td>45.82</td>
</tr>
</tbody>
</table>

Table 5. Coefficient result of fitting of General model Gaussian.

<table>
<thead>
<tr>
<th>Mathematical function</th>
<th>Coefficient results after fitting</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = a_1 \exp(-((x-b_1)/c_1)^2) )</td>
<td>( a_1 = 303 ) ( b_1 = 10.51 ) ( c_1 = 1 )</td>
</tr>
</tbody>
</table>

Oversold Model Establishment Based on Flight Departure Time Distribution

**Basic Assumptions.** Models are built based on the following assumptions:

(1) The time distribution of passenger arrivals is roughly consistent with the time distribution of aircraft departures; (2) The passenger’s reservation request occurs independently; (3) The travel needs of passengers are unlimited; (4) All passengers who have been denied boarding due to overbooking by the airline will still choose to continue their journey; (5) Does not consider the impact of multi-class cabins.

**Model Establishment.** (1) Symbol description

- \( c \): the actual seat number of the aircraft, that is the aircraft capacity;
- \( r \): Represents the net profit obtained by the company after the ticket is sold to each passenger;
- \( b \): indicates the compensation fees that the airline handles for a passenger who cannot board the plane because the airline has oversold;
- \( x \): indicates the number of seats sold on a certain flight;
- \( u \): indicates the arrival rate of passengers;
- \( f(u) \): Probability density function representing passenger arrival rate \( u \);
- \( TC \): indicates the total oversold cost of a flight.

(2) Model establishment

When \( x \leq \frac{c}{u} \), due to the low passenger arrival rate on the flight, the actual number of people boarding could not fully use the flight location, the vacancy for the airline is lost. The oversold cost is:

\[
TC = r \left( c - ux \right) \quad (3-1);
\]

When \( u \geq \frac{c}{x} \), the passenger arrival rate is large at this time, and the actual number of people boarding exceeds the actual capacity of the aircraft. In order to ensure the normal departure of the flight, some passengers will be compensated for being denied boarding due to overbooking by the airline. The oversold cost is:

\[
TC = b \left( ux - c \right) \quad (3-2);
\]

So the average cost of oversold aircraft can be written as:

\[
TC = \int_0^x r(c-ux)f(u)du + \int_\frac{c}{x}^1 b(ux-c)f(u)du \quad (3-3);
\]

Through differential deformation, the final derivative is obtained:

\[
\int_0^\frac{c}{x} uf(u)du = \frac{b}{r+b} \int_0^1 uf(u)du \quad (3-4).
\]

**Case Study and Model Results.** taking a nanjing-xiamen flight as an example, according to the experience of previous airlines, after each passenger purchasing a ticket, the company’s income is 900 yuan / piece. The model of this flight is A320 and the actual passenger capacity of the flight is 160. When some passengers are denied boarding due to overbooking, the compensation fee is approximately 200 yuan. It can be known from the foregoing calculation results that after
normalizing the passenger trip rate \( u \) in the morning period, it follows the normal distribution of the parameter \( \sim N (\mu, \sigma) = N (0.95,) \).

It is directly solved according to formula (3-4). It is difficult to obtain an accurate solution of the variable upper limit integral equation. This paper uses MATLAB mathematical software to solve the above equation in two steps. The first step is to substitute the data to find the result on the right side of the equation

\[
\int_{0}^{900+200} uf(u)du = \frac{200}{1100} \times 0.8478 = 0.1541.
\]

The second step is to bring into the integral equation

\[
\int_{x}^{160} uf(u)du = 0.1541,
\]

and then the approximate solution \( x = 170 \) of the actual seat number sold on the flight is obtained.

In summary, the research results show that when the passenger arrival rate \( \sim N (\mu,) \) is unchanged, if the relationship between the passenger arrival rate \( \mu \), the best flight ticket sales and passenger arrival rate \( \mu \) is changed, it is easy to find that the larger passenger arrival rate is \( \mu \), the lower the best flight ticket sales is. This is because when the passenger's arrival situation is very close to the previous booking situation, the airline only needs a small amount of overbooking to ensure the effective use of flight seats. On the contrary, excessive overbooking will reduce the total profit for the increase of DB costs.

**Conclusion**

Based on the distribution of airline flight departure time, an oversold model centered on passenger arrival rate was established. The research results show that the model assumptions are consistent with the actual situation. The calculation basis for airline oversold revenue management is an oversold model based on the passenger arrival rate. The passenger arrival distribution curve described by this model is the time distribution curve obtained by shifting forward the flight departure time distribution curve. By analyzing the model is verified correctly and effectively.

**Acknowledgement**

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**References**


