Dynamics Model of Backhoe Devices Considering Excavation Load for Hydraulic Excavators

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Abstract. This paper presents a new dynamic model of backhoe devices considering excavation load so as to study the dynamic characteristics when the backhoe device is excavating. The dynamic model for the backhoe device was derived in matrix form by using Lagrange equation. The program of fourth-order Runge-Kutta method was coded in MatLab software so as to solve the dynamic model. Then taking composite excavation conditions as examples, The proposed dynamic model was used to calculate the counterforce of the kinematic pairs at all hinge points and solved the rotational movement laws (such as angular displacement, angular velocity, etc.) of the main components of backhoe devices around their rotary axes. In order to verify the correctness of the dynamic model, experiments of composite excavation conditions were done by using an XG822FL excavator, and the results of the experiment demonstrated that the proposed dynamic model is correct and effective to calculate the counterforce of kinematic pairs and solve the rotary movement laws of main components during the course of excavation, which can be used to predict the counterforce of kinematic pairs during the design process of backhoe device.

Introduction

Hydraulic excavator, which is one of the most widely used construction machineries, has complex structure and typical functions. It plays an important role in transportation, mining, construction industry, agriculture, forestry and earthquake relief engineering. A backhoe device, which is one of main working parts of hydraulic excavator and consists of a boom, an arm, a bucket and three groups of driving hydraulic cylinders (shown in Fig.1), has very complex forces when it is excavating. In order to study the mechanics characteristics (such as strength, stiffness and vibration) of the backhoe device during excavation, it is necessary to establish the excavating dynamic model of the backhoe device. However, the dynamics model of the backhoe device, belonging to the domain of multibody system dynamics, is very difficult to be established. At present, the methods commonly used in engineering to establish multibody system dynamic model include analytical mechanics represented by Lagrange equation [1], vector method represented by Newton-Euler equation [2-4], Kane method [5-6] and variational method [7].

The dynamical modeling of backhoe devices of excavator has attracted little attention in the literature. Without considering the degree of rotation freedom, the backhoe device of excavator is a three-degree-of-freedom parallel mechanism, which can realize various complex excavation operations. Some scholars had studied the dynamics model of the backhoe device of excavator by Newton-Euler method. Koivo [8] built the dynamic model of excavator, which describes the system motion with time, for automation control by using Newton-Euler method. Sun [9] established the dynamics model of excavator working device by Newton-Euler method, and verified the model with an example. B. Fox [10] also use Newton-Euler method to establish system dynamics model of
excavator. However, the Newton-Euler method is used to establish the dynamics model of multibody system, involving complicated analysis of force and motion. In order to obtain the dynamic equation of multibody system, the force and motion analysis of each component in the system must be carried out, and its dynamic models must be established respectively. The whole process is complicated, time-consuming and error-prone.

In this paper, a new dynamics model of the backhoe device considering excavation load is put forward by using Lagrange method, which can describe dynamic characteristics of the backhoe device in the excavation. During the derivation process of dynamics model, the total kinetic and potential energy of backhoe device is calculated. Meanwhile, the force and motion analysis of backhoe devices is observed in the course of excavation so as to calculate the net torque vector acting on the joints. The dynamic model of backhoe devices is obtained by derivation and calculation from Lagrange equation. Then the proposed dynamic model is applied to analyze the dynamic characteristics when the backhoe device is working on composite excavation condition and leveling condition.

Establishing Dynamics Model of Backhoe Devices for Excavators

Lagrange Equation of Backhoe Devices

The backhoe device of excavators, which consists of a single DOF boom mechanism ABC, a single DOF arm mechanism DEF and a single DOF bucket mechanism GMKQN, is a three-degree-of-freedom planar mechanism without considering the degree of freedom of rotation as shown in Fig. 1. In order to simplify the problem analysis and build dynamical model, the backhoe device should be properly abstracted. So the boom cylinder, arm cylinder and bucket cylinder are removed by replacing them with the net driving torque $T_i (i = 1, 2, 3)$ varying with time. What’s more, the specific shapes of the boom, arm and bucket are neglected and replaced them by lines $CF, FQ, QV$ respectively, placing their equivalent masses $m_i (i = 1, 2, 3)$ at their respective centroid positions and their equivalent moment of inertia $I_i (i = 1, 2, 3)$ acting on their respective axis of rotation. Consequently, the dynamics model of backhoe device with three generalized coordinates $\theta_i (i = 1, 2, 3)$ is obtained as shown in Fig. 2.

![Figure 1. Backhoe device of excavators.](image1)

![Figure 2. Dynamics model of backhoe devices.](image2)

According to the dynamics model shown in Fig. 2, the Lagrange function of excavator backhoe device is defined as:

$$L = K(\Theta, \dot{\Theta}) - P(\Theta)$$

(1)

where $\Theta = [\theta_1, \theta_2, \theta_3]^T$ represents the generalized coordinate vector, $\theta_i (i = 1, 2, 3)$ is the generalized coordinates for the dynamics model of backhoe devices. $K(\Theta, \dot{\Theta})$ represents the total kinetic energy of all components of the backhoe device. $P(\Theta)$ represents the total potential of all components of the backhoe device.

The Lagrange equation of backhoe devices is given by
where $T$ represents the net torque vector acting on the joints, which is given by $T = [T_1, T_2, T_3]^T$.

**Determination of the backhoe device’s $K(\Theta, \dot{\Theta})$, $P(\Theta)$, $T$**

$K(\Theta, \dot{\Theta})$ represents the total kinetic energy of all components of the backhoe device. It can be expressed as the sum of translational kinetic energy and rotational kinetic energy, and given by

$$K(\Theta, \dot{\Theta}) = \frac{1}{2} v^T M v + \frac{1}{2} \omega^T I \omega$$

where $M$ represents the equivalent mass matrix of components, which is a diagonal matrix and is given by $M = \text{diag}[m_1, m_2, m_3]$, $v$ represents the centroids velocity vector of components, which is given by $v = [v_{c1}, v_{c2}, v_{c3}]^T$, $\omega$ represents the angular velocity vector of components about their respective axis of rotation, which is given by $\omega = [\omega_1, \omega_2, \omega_3]^T$, $I$ represents the rotary inertia matrix of components about their respective axis of rotation, which is diagonal matrix and given by $I = \text{diag}[I_1, I_2, I_3]$.

$P(\Theta)$ represents the total potential energy of all components of the backhoe device, which is given by

$$P(\Theta) = MH_v \cdot g$$

where $H_v$ represents the centroids height vector of components, which is given by $H_v = [h_{c1}, h_{c2}, h_{c3}]^T$, $g$ represents the gravity acceleration.

$T$ represents the vector of net torques acting on the joints, and is given by $T = [T_1, T_2, T_3]^T$. What’s more, $T$ is also calculated by

$$T = T_g - T_L$$

where $T_g = [T_{g1}, T_{g2}, T_{g3}]^T$ represents the driving torque vector of the cylinder to the corresponding joint points, $T_L = [T_{l1}, T_{l2}, T_{l3}]^T$ represents the equivalent resisting torques vector generated by excavation load at each of joints. In order to gain the value of $T_g$ and $T_L$, the force analysis diagram of the backhoe device is drawn when it is excavating, as shown in Fig. 3. From the force analysis diagram, $T_g$ and $T_L$ are easy to be calculated according to the following equations.

$$T_L = \begin{bmatrix} T_{l1} \vspace{1em} \\
T_{l2} \vspace{1em} \\
T_{l3} \vspace{1em} \end{bmatrix} = \begin{bmatrix} F_{1l} \vspace{1em} \\
F_{2l} \\
F_{3l} \vspace{1em} \end{bmatrix}$$

$$T_g = \begin{bmatrix} T_{g1} \vspace{1em} \\
T_{g2} \vspace{1em} \\
T_{g3} \vspace{1em} \end{bmatrix} = \begin{bmatrix} F_{1g} \vspace{1em} \\
F_{2g} \\
F_{3g} \vspace{1em} \end{bmatrix}$$

where $F_t$ denotes the tangential force acting on the tip of bucket tooth, $F_n$ denotes the normal force acting on the tip of bucket tooth, $F_i (i = 1, 2, 3)$ denotes the thrust force of hydraulic cylinder, and
\[ F_i = pS_i \ (i = 1, 2, 3) \], where \( p \) is the working pressure of hydraulic system, \( S_i \) is the piston area of hydraulic cylinder. \( l_{**} \) denoting the distance between the corresponding two points is the arm of corresponding force.

**Dynamics Model of Backhoe Devices**

Substituting Eq.1 into Eq.2 and combining the Eq.3, 4 and 5, we can obtain the dynamics model of backhoe devices considering excavation load, which is given by

\[
\frac{\partial^2 K}{\partial \Theta \partial \Theta} \ddot{\Theta} + \frac{\partial^2 K}{\partial \Theta \partial \Theta} \dot{\Theta} + \frac{\partial^2 K}{\partial \Theta^2} + \frac{\partial P}{\partial \Theta} = T
\]

(8)

If let \( J(\Theta) = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \), representing the equivalent rotation inertia matrix, where

\[ J_{ik} = \frac{\partial^2 K}{\partial \dot{\theta}_i \partial \dot{\theta}_k} \ (i = 1, 2, 3 \ , \ k = 1, 2, 3) \]. \( J(\Theta) \) is a symmetric matrix. If \( J_{ik} \neq 0 \ (i \neq k) \), the Eq.8 has dynamical coupling.

If let \( C(\Theta) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \), representing the coupling array of joints angular velocity, where \( C_{ij} = \frac{\partial^2 K}{\partial \dot{\theta}_i \partial \dot{\theta}_j} \ (i = 1, 2, 3 \ , \ j = 1, 2, 3) \). \( C(\Theta) \) is also a symmetric matrix. If \( C_{ij} \neq 0 \ (i \neq j) \), the Eq.8 has velocity coupling.

If let \( Q(\Theta) = \begin{bmatrix} Q_1(\Theta) \\ Q_2(\Theta) \\ Q_3(\Theta) \end{bmatrix} \), representing the generalized force vector. So the Eq.8 is expressed by

\[ J(\Theta) \dddot{\Theta} + C(\Theta) \ddot{\Theta} + Q(\Theta) = T \]

(9)

Eq.9 is the dynamics model of backhoe devices considering excavation load, which is a second order differential equation concerning the joint variables. This is a matrix equation that is simple and easy to read, but is highly abstract and inconvenient to calculate. Expanding the matrix, Eq.9 is also expressed by
Application of the Dynamics Model

The dynamic model of backhoe device can be used to solve the following three kinds of basic problems: the first kind of problem is to calculate the forward dynamic solution to obtain the law of backhoe device’s movement on condition that the external force acting on the backhoe device is known; the second kind of problem is to solve the inverse dynamic solution to obtain the external force acting on the backhoe device on condition that the law of backhoe device’s movement is known; and the third kind of problem is the synthesis of the above two kinds of problems, that is to say, the third kind of problem is a hybrid problem of forward and inverse dynamic solutions. Next, taking an XG822FL excavator of Xiamen construction machinery as an example, the dynamic characteristics of backhoe devices is analyzed.

Dynamic Behavior of the Backhoe Device under Composite Excavation Conditions

Composite excavation usually requires the combined movement of arm and bucket to complete. Under the composite excavation condition, it is usually known that the external load (excavation resistance force $F_l$ and hydraulic cylinder driving force $F_g$) and the dynamics initial conditions (as shown in Table 1). So the dynamic model is applied to solve the movement law of backhoe device and calculate the counterforce of the kinematics pairs.

<table>
<thead>
<tr>
<th>components</th>
<th>$\theta_0$ (deg)</th>
<th>$m_i$ (kg)</th>
<th>$I_i$ (kg·m²)</th>
<th>$l_i$ (m)</th>
<th>$\dot{\theta}_0$ (deg/s)</th>
<th>$\beta_i$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>doom</td>
<td>43.5</td>
<td>913</td>
<td>819</td>
<td>2.551</td>
<td>0</td>
<td>18.3</td>
</tr>
<tr>
<td>arm</td>
<td>308.9</td>
<td>591</td>
<td>595</td>
<td>1.466</td>
<td>0</td>
<td>13.4</td>
</tr>
<tr>
<td>bucket</td>
<td>292.7</td>
<td>427</td>
<td>353</td>
<td>0.562</td>
<td>0</td>
<td>32.7</td>
</tr>
</tbody>
</table>

where $\theta_0$ represents the initial angle of each component, $\dot{\theta}_0$ represents the initial angular velocity of each component, $l_i$ represents the length of each component from the centroid to the corresponding joint, $\beta_i$ represents the angle of each component from the centroid to the corresponding center line. All the parameters are shown in Fig. 2.

Excavation force refers to the actual force acting on the cutting edge of the bucket when the arm cylinder or the bucket cylinder operated alone. Excavation process can be divided into bucket digging, arm digging and composite digging according to the different excavating movement of backhoe device. The composite digging is usually adopted in the actual operation process of excavator, so it is analyzed in detail here. According to the reference [11], the excavation resistance force $F_t$ during the process of composite excavation can be calculated according to equation as follows.

$$
\begin{align}
F_t &= K_0bh \\
F_n &= \varphi F_t \\
F_i &= \sqrt{F_t^2 + F_n^2}
\end{align}
$$

where $K_0$ denotes the excavation resistance force per unit area, which is related to soil properties and section shape. Unit: $N/cm^2$. $b$ denotes the average width of the bucket. $h$ denotes the depth of cut. $\varphi$ denotes the coefficient of excavation resistance. For Class III soil, the mean tangential load spectrum of bucket tooth tip during composite excavation is obtained experimentally, as shown in Fig. 4. The value of $F_i$ is gotten in different time by sampling the mean tangential load spectrum, and the value of
$F_n$ and $F_i$ is also gotten according to the Eq.11.

$T_i$ is obtained by substituting $F_i$ and $F_n$ into the Eq.6. $T_g$ is also obtained by substituting $F_g=[F_1 \ F_2 \ F_3]^T$, which is the thrust force of each hydraulic cylinder acting on the boom, arm and bucket, into the Eq.7. Then the net torque vector $T$ is calculated according to Eq.5.

When the net torque vector and the initial conditions of the backhoe device are known, the dynamics model is applied to solve the movement law of the backhoe device and calculate the counterforce of the joint kinematic pairs, which is the preparation for the structural design of the main structural parts (boom, arm and bucket) of the backhoe device. Next, the solving process of dynamics problem is discussed in detail.

**Solution of the Dynamics Problem**

From the above discussion, it can be seen that the dynamics model is applicable regardless of the working conditions of backhoe device. In this section, the solution of the dynamics model will be discussed.

The dynamics model is a system of second-order differential equations, which can be solved by many methods, such as decoupling method, eigenvalue method and numerical method [12-13]. In this paper, the four order Runge-Kutta method, a classical method to solve differential equations, is used to solve the dynamics model. In order to solve the dynamics model by using the four order Runge-Kutta method conveniently, the dynamics model is rewritten into an iterative form and expressed by matrix as iterative equation.

$$
\ddot{\Theta} = J^{-1}(\Theta) \left[ T - C(\Theta) \dot{\Theta} - Q(\Theta) \right]
$$

where $J^{-1}(\Theta)$ is the inverse matrix of $J(\Theta)$. Subsequently, the program of fourth-order Runge-Kutta algorithm is compiled in MatLab, which is used to solve the Eq.12 in a certain time.

Taking the composite excavation conditions discussed above as examples, when the mean value of the net torque $T$ acting on each joint under the combined excavation is known and the dynamics initial conditions are known, as shown in Table 1, the curves of the counterforce of main hinge pivots varying with time are obtained by solving the Eq.12, as shown in Fig. 5. Further more, the curves of the angular displacement, angular velocity and angular acceleration of main components (such as the boom, arm and bucket) varying with time are also obtained by solving the Eq.12, as shown in Fig. 6.

![Counterforce curves of hinge points in the boom](image1)

![Counterforce curves of hinge points in the arm](image2)
Verification of the Dynamics Model of Backhoe Device

In this experiment, an XG822FL excavator of Xiamen construction machinery was used to carry out composite excavation test. The Multi-system 5060 tester of Hydrotechnik Company, PTS410 strain pressure sensor and KTJV010 angular displacement sensor of Southern Measurement and Control Equipment Factory were used to carry out the experiment. The purpose of the experiment is to verify the correctness of the dynamics model.
From the above section analysis, we can know that the dynamics model describes the main dynamic behavior and characteristics of backhoe device. There are a lot of test points to verify the dynamics model of backhoe device. If all points selected to test, the process of test operation is tedious. So this experiment selected the hinge point C (shown in Fig.1) to test the counterforce of kinematic pair and the boom (shown in Fig.1) to test the angular displacement. The variation curves of counterforce in hinge point C and angular displacement of boom under composite excavation conditions were obtained by experimental test, and then compared them with the corresponding variation curves obtained by the dynamics model, as shown in the Fig.7.

![Figure 7. Comparison between experimental results and theoretical results.](image)

**Discussions**

It can be shown from Fig. 5 that the counterforce of kinematic pairs at all hinge points obtained from the proposed dynamic model, in the whole, are not large before excavation. Only the counterforce of the joint between the boom and the boom cylinder is larger than other joints' before excavation because the boom lifts the whole backhoe device to adjust the position of excavation. During the excavation, the counterforces of the kinematic pairs at all the hinge points increase and reach the maximum. The hinge points in the boom have the maximum counterforce, which reaches 220kN approximately, while the hinge points in the bucket have the minimum counterforce. After the bucket passes the maximum cutting depth, the counterforces of all hinge points begin to decrease, but they are greater than the counterforces before excavation because the bucket is fully loaded. When the bucket is unloaded, the counterforces of all hinge points are back to the level before excavation.

It can be shown from Fig. 6 that the boom and arm rotate slowly and smoothly around their hinge points, and the angular velocity and angular acceleration are not large. But the bucket rotates around the hinge point frequently and its angular velocity is faster. Sudden state changes cause a relatively large angular acceleration and a relatively large impact vibration. Therefore, the hinge shafts need to be replaced periodically because of wear or fracture caused by shock and vibration when they works, but the replacement frequency of shafts on the bucket is faster than that on the boom and arm. This is consistent with the actual use of the backhoe device.

It can be shown from Fig. 7 that the counterforce of the kinematic pair at the testing hinge point is basically the same as that obtained from the proposed dynamic model. The counterforce obtained from the experiments is less than that obtained from the dynamic model, and the maximum error is less than 10 percent. Further more, the angular displacement of the boom obtained from the experiments basically agrees with that obtained from the dynamic model, but it is ahead of about 2 seconds than that obtained from the dynamic model, which may be related to the calibration and reflection time of the angular displacement sensor used in the experiment.

From the above discussion shows that the proposed dynamics model of backhoe devices, which will be used to predict the dynamic characteristics of backhoe device during the excavation, is verified to be correct and effective by the experiments. Although there is a little difference between the results
of the dynamic model and that of the experiments, the errors are acceptable because they are within the allowable range of projects.

Conclusions

The paper has presented a new dynamics model of backhoe devices considering excavation load based on the Lagrange method. The dynamics model can describe dynamic characteristics of backhoe devices during the process of excavation. In order to establish the dynamics model, the total kinematic energy and potential energy of the backhoe device have been calculated firstly, and then the force and motion analysis of the backhoe device is observed in the course of excavation so as to calculate the net torque vector acting on the joints. Finally the dynamic model of backhoe devices, which reflects the dynamic behavior of backhoe devices, is derived in matrix from Lagrange equation.

Then taking composite excavation condition and leveling condition as examples, the proposed dynamics model is applied to calculate the counterforce of all kinematic pairs at the hinge points and solve the rotary motion laws of the boom, arm and bucket. In order to verify the proposed dynamic model correctness, a composite excavation experiment has been done by using an XG822FL excavator of Xiamen construction machinery. The results of experimental test agree with the theoretical results of the dynamic model, which has been proved that the proposed dynamic model is correct and effective to describe dynamics characteristics of the backhoe device.

The proposed dynamics model not only describes the dynamic characteristics of backhoe devices and solves the forward and inverse dynamics problems, but also is the basis for automatic control, trajectory planning and optimization design of backhoe devices. Further more, it is also the basis for structural design and strength analysis of backhoe devices because it can be used to calculate the counterforce of kinematic pairs of backhoe devices under various loads and motion states, which is of great significance to improve the design level of excavators.

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References


