Simulations of Optimizing Strategies for Forming of Mutual Insurance Companies Fund

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Abstract. Mutual insurance companies are subject to high risks of ruin due to lack of funds. Thus, the problem of optimizing strategies for forming the company’s fund are being actualized, ensuring an increase in its financial stability under assumed random insurance payments flow by rationalizing the size of initial and current contributions of its participants, the volume and time parameters of asset securitization, loans taken, insurance risk taxes taking into account the possibility of obtaining subsidies, reinsurance of risks and some other forms of the state support. The criterion in such task is the minimization of probabilities of the company’s ruin. As restrictions costs of policyholders and the state can be considered but those determined by legislation. Because of the random nature of many indicators and the complex relations among them, it is rather difficult to obtain a solution to such optimization problems through analytical methods. For these purposes, simulations are preferred.

Introduction and Related Work

The insured, united in the framework of the mutual insurance company (MIC) and being the joint owners of its fund, under certain conditions can get more profitable insurance coverage than with commercial insurance.

However, to a large extent, this assertion is true for MIC with participants that are homogeneous in nature and level of risks and have a long history of their activity, which enabled them to accumulate significant funds [1]. New MIC, whose funds are formed through initial contributions of policyholders and then filled up due to their current payments, at the beginning of their activities are characterized by high risks to ruin, which reduces their attractiveness for policyholders [2, 3]. This, to some extent, explains the lack of development of the mutual insurance market in the Russian Federation, where only 12 MIC are registered at the moment, and judging by the turnover, only one of them is active: the “MIC of Civil Liability of Builders”, uniting almost 350 participants.

It is possible to rectify the current situation, in our opinion, by the use of the mechanisms for accelerated filling of the funds of the MIC at the beginning of their activities in the framework of mixed financing schemes with the involvement of external sources and, in particular, within the framework of programs to support certain types of insurance from state and regional budgets [4].

On a contrary, such schemes are characterized by different efficiency, which implies the expediency of choosing and justifying the most rational options [5].
This article discusses approaches to solving this problem by optimizing possible strategies for managing the formation of a MIC fund with a homogeneous composition of insurers, given a given level of financial stability, according to the criteria characterizing the attractiveness of their participation in the society, taking into account the possibilities of mixed financing [6]. A significant variety of such strategies and the random nature of the financial flow processes under consideration make it difficult to obtain an analytical solution to this problem by stochastic analysis and optimization methods due to the complexity of computations and the complexity of calculations. More effective in this situation are the methods of simulation used in this study.

The Process of Accumulation of the MIC Fund and the Indicators of its Stability

Suppose that the MIC is formed by \( N \) homogeneous participants, each of which in the year \( t \) is characterized by a random individual risk of losses \( q^t_n \), where \( n = 1, N; t = 1, T \). In order to simplify further calculations, we will consider these random variables to be independent and identically distributed over the community and time.

We introduce the following assumptions regarding the order of the operation of the MIC for \( T \) years.

1. The MIC during the \( T \) years is formed by annual payment made by each participant a certain insurance premium \( r^t_n = r^t \), which is the same for all participants due to the homogeneity of the risks. The value in addition to the current insurance premium includes also the entrance fee of the participant.

2. There are no external sources of replenishment of the fund.

3. The MIC bears full responsibility for losses \( q^t_n \).

4. In order to maintain the activity of the MIC each year \( t \) certain expenditures \( C^t \) are allocated.

In the framework of the model under consideration, we will consider the level of expenditure to be constant in time \( C^t = C \).

5. After the expiry of \( T \) years, the funds accumulated by the fund are returned to the insured in equal shares.

Taking into account these assumptions, the balance of the fund’s resources \( H^t \) after \( t \)-th year of activity \( t \leq T \) is determined by the following expression:

\[
H^t = \sum_{\tau=1}^{t} N \cdot r^\tau - \sum_{\tau=1}^{t} Q^\tau - t \cdot C, \tag{1}
\]

where the value \( Q^\tau = \sum_{n=1}^{N} q^\tau_n \) represents the total amount of payments in the year \( \tau \).

Sequence of values \( \{H^t\}_{t=1,T} \) is a random process of accumulation of funds by the MIC fund, controlled by parameters \( \{r^t\}_{t=1,T} \).

Let the random variable \( \tau^* \) characterize the moment of the alleged ruin of the MIC, which can be defined as the smallest value of \( t \) at which the amount of the insurance fund’s resources turns out to be negative:

\[
\tau^* = \inf\{t : H^t < 0\}. \tag{2}
\]

We note that the inequality \( H^t < 0 \) determines the condition of the technical ruin of the MIC fund, characterized by the lack of actual financial resources necessary for the implementation of further
insurance activities. Therefore, in the expression (1), the effects of a change in the value of the different cash flows, usually reflected by the corresponding discounts, are not taken into account.

As a measure of the stability of the MIC, consider the probability of its fund being not ruin during a period \(T\), which can be defined as the probability of exceeding \(T\) the value of \(\tau^*\):

\[
P = \Pr\{\tau^* > T\}.
\]

(3)

The effectiveness of the participation in the MIC of a separate insured, who is making a decision at a time \(t = 0\), will be estimated by an indicator characterizing the ratio of the mathematical expectations of its costs to the policyholder’s company for \(T\) years and losses in the absence of insurance. At the same time, we will take into account that the policyholder takes into account the effects of a decrease in the value of financial assets in time, which are valued at a discount \(d\) equal to the profitability of its capital \([7]\).

In the absence of insurance, the aggregate loss of the insured for \(T\) years, estimated taking into account discounting, is a random value determined by the following expression:

\[
S_0^n = \sum_{t=1}^{T} \frac{q_t^n}{(1 + d)^t}.
\]

(4)

The expenses of a participant in a MIC under conditions of a random nature of payments are a random variable, the value of which depends on the results of the entire MIC as a whole, taking into account the possibility of the ruin of the latter. We assume that for the insured there is a certain acceptable level of risk of ruin \(\alpha\), at which achievement he neglects the possibility of such an event.

The conditional expected value of the participant’s expenses in the event that the MIC fund is not ruined during a period \([0, T]\) can be estimated as the difference in the contributions made and the stake of the participant in the balance of the insurance fund at the time \(T\) estimated by its expected value with discounting:

\[
E[S_n|\{\tau^* > T\}] = \sum_{t=1}^{T} \frac{r^t}{(1 + d)^{(t-1)}} - \frac{E[H_T|\{\tau^* > T\}]}{N \cdot (1 + d)^T}.
\]

(5)

The reduction in the degree of the denominator per unit in the first term reflects the fact that the insurance premium is paid before the beginning of the next year.

Taking into account (4) and (5), the indicator of the effectiveness of participation in the MIC of a particular insured is determined by the following expression:

\[
u = u_n = \frac{E[S_n|\{\tau^* > T\}]}{E[S^0_n]}.
\]

(6)

The ratio (6) expresses the relative level of overpayment for risk insurance in the MIC. Its value differs depending on the selected mode of filling the fund of MIC and, as a result, it can be used to compare the efficiencies of various options for this process.

**Optimal Control of MIC at its Isolated Work**

The objective of optimal management of the process of accumulating the fund of MIC \([1]\) in the absence of co-financing and other types of support can be formulated as follows: it is necessary
to determine the strategy for the formation of the fund through initial and current contributions \( \{r^t\}_{t=1,T} = \overrightarrow{r} \), in which the cost of insurance protection for participants is minimal, sustainability of the fund P:

\[
\begin{cases}
    u(\overrightarrow{r}) \to \min_{\overrightarrow{r}} \\
    P(\overrightarrow{r}) \geq 1 - \alpha \\
    r^t \geq 0
\end{cases}
\] (7)

It can be shown that, in view of the monotonicity of the functions \( u(\overrightarrow{r}) \) and \( P(\overrightarrow{r}) \) for each of the arguments, the solution of problem (7) exists and is located on the surface \( O = P^{-1}(1 - \alpha) \) (on the boundary of the admissible region).

At the same time, analytic expressions of dependencies \( P(\overrightarrow{r}) \) and \( u(\overrightarrow{r}) \) are difficult to form in practice. As a result, in solving problem (7) it is proposed to use methods of simulation [8].

Simulation experiment will be put as follows: for each of the \( N \) participants of the society we will simulate the flows of random losses \( q^t_n \), on the basis of which we will generate samples of realizations of the accumulation process of the MIC fund \( \{1\} \) for \( t \in [1,T] \) with fixed external parameters \( N, q^t_n, C \) and fixed control parameters \( r^t \). On the basis of these samples, the probability of fund to ruin \( P \) is defined as the ratio of the number of realizations of the process \( \{H_t\} \) that do not contain values, which are \( H_t < 0 \), to the total number of conducted experiments. The conditional mathematical expectation \( H_T \) is calculated as the average value of the value taken for these realizations, and the expected value of the insured’s losses for \( T \) years in the absence of insurance \( E[H_T|\{\tau^* > T\}] \) is estimated as the average value of the exponent \( S_0^n \), taken over all the realizations of the losses \( q^t_n \). Taking into account these initial and calculated indicators and discount \( d \), the value of the criterion \( u(r) \) is determined by expression (6).

Then, using the described algorithm for obtaining the values of the functions \( P(\overrightarrow{r}) \) and \( u(\overrightarrow{r}) \), for varying values of the weights \( r^t \) by the method of gradient descent, we obtain a numerical solution of the problem (7) for a fixed value of the parameter \( \alpha \).

In carrying out simulation experiments, the loss streams of the participants in the MIC were formed as follows: losses of the \( n \)-th agent in the \( t \)-th year \( q^t_n \) take place with probability \( p \), and their value is determined by some random value of \( \pi \). As the probability distribution of the latter, without loss of generality, it is proposed to use the exponential distribution characteristic for many cases. Then the characteristics of individual risk can be determined as follows:

\[
q^t_n = \theta \cdot \pi, \quad \theta = \begin{cases} 1, & p \\ 0, & 1 - p \end{cases}, \quad p_{\pi}(x) = \lambda \cdot e^{-\lambda x}, \quad x \geq 0.
\] (8)

In each conducted experiment, \( 10^5 \) implementations of the model were considered with the distribution parameters \( q^t_n \), which are equal to \( p = 0.1 \) and \( \lambda = 1 \). As the unit of measure for the amount of money, the expected payments were taken for one occurred insurance event \( E[q^t_n|\theta = 1] = \lambda^{-1} = 1 \).

In all experiments, the number of agents was chosen equal to \( N = 50 \), the period of the MIC activity was \( T = 10 \) years. The discount rate in the basic experiment was taken equal to 0.2, the cost of maintaining the activity of the MIC is \( C = 0 \). The permissible level of ruin probability in most experiments is in the range of 5 to 25% (using higher levels of stability involves a significant increase in the sample size necessary for calculating the quantities considered).
Results of the Simulation Experiment

The graphs reflecting the numerical solutions of the optimization problem at different levels of the permissible probability of ruin $\alpha$ are shown in the left-hand part of Fig. 1. They indicate that for the initial data considered the best strategy for ensuring the stability of the MIC for its participants is to make significant funds (2-3 times higher than the expected annual payments) at the very beginning of its activity, with a relatively uniform annual replenishment of the fund in the future. Under this strategy, the level of the insurance premium, starting from the second year of the MIC activity, is on the average in the range from 1.2 to 1.4 of the expected annual payments, depending on the chosen level of the fund’s stability $\alpha$.

Figure 1. Optimal strategies for the MIC fund formation at various levels of stability (on the left) and discount rates (on the right)

The dynamics of the premium value is largely determined by two opposite consequences of replenishment of the fund. Early contributions to the fund are more effective in terms of sustainability, but increase the cost of insurance because of the discount effect. The right-hand side of Fig. 1 shows the solutions of problem with an allowable probability of ruin of the MIC $\alpha = 15\%$ for different discount levels $d$. They show that the optimal premium size can both decrease for small values of $d$ (the moment of investment is indifferent), and locally grown at high (delay of investments is preferable).

The left part of Fig. 1 shows the dependence of the cost of participation in the MIC on the required level of stability when using the standard scheme of fund formation with a constant insurance premium level and the optimal schemes obtained as a result of solving the problem. From the above graph, it follows that the use of a more complex procedure for filling the fund allows the insured to save up to 20\% of the cost of insurance while maintaining the same level of stability. In general, the exponential nature of the dependence of the cost of insurance on the required stability, described in [9], is also preserved in the solutions of the optimal control problem.

The relative cost of insurance increases with the increase in the discount factor: at high levels of return on equity, it is more advantageous for policyholders to take the risk than to deposit income-bearing funds into the insurance fund (solid line in the right part of Fig. 2). At low levels of profitability of own funds, the relative cost of insurance $u$ may be less than unity (because of the
exclusion from the calculation of cases of the ruin of the fund). In this case, from formal positions, participation in the MIC can be considered as a risky investment tool.

Figure 2. Dependencies of the minimum cost of insurance on the level of stability (on the left) and discount rate (on the right)

Introduction to the model of fixed costs $C$ for the maintenance of the activity of MIC does not affect the shape of the curves described, evenly increasing the cost of insurance and the level of premiums (dashed lines in the right part of Fig. 2). Given the level of stability of the fund, the dependence of the cost of insurance on the value of fixed expenses with a high degree of accuracy can be described by the following approximating expression:

$$u(C) = u(0) + \frac{C}{N \cdot E[S^0_t]} \cdot \sum_{t=1}^{T} \frac{1}{(1 + d)(t-1)}.$$  

Optimal Management of MIC in Conditions of Mixed Financing

Suppose that the MIC throughout the review period has the ability to freely exchange financial flows $f^t$ with some third-party organization (Center), which has a rate of return on its own funds $g \neq d$.

We assume that the positive values correspond to the cash flows from the Center to the MIC, and the negative values from the MIC to the Center. Then the equation of the process of accumulation of the MIC fund [1] can be transformed to the following form:

$$H^t = \sum_{\tau=1}^{T} (N \cdot r^\tau + f^\tau) - \sum_{\tau=1}^{T} Q^\tau - t \cdot C,$$  

The net discounted income of the center from participation in the financing of the MIC, in the event of a fund failure, will amount to:

$$G(f) = \sum_{t=1}^{T} \frac{-f^t}{(1 + g)(t-1)}.$$  

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We assume that the Center takes a decision on the financing of the MIC if its income is greater than a certain constant value $G(\vec{f}) \geq G_0$. In the case of $G_0 > 0$ the Center, it can be interpreted as a commercial financial institution. The case $G_0 < 0$ is possible if the role of the Center is an agent pursuing any non-commercial interests. In particular, such an organization can be a state that provides support to MIC.

Given the availability of an additional source of funding, the optimization problem (7) can be modified as follows:

$$\begin{cases}
u(\vec{r}, \vec{f}) \rightarrow \min \\
P(\vec{r}, \vec{f}) \geq 1 - \alpha \\
G(\vec{f}) \geq G_0 \\
r^t \geq 0
\end{cases},$$

where $\vec{f} = \{f^t\}_{t=1,T}$.

The criterion (12) takes into account that flows $f^t$ implicitly enter the expression for the cost of insurance $u(\vec{r}, \vec{f})$ through the component $E[H^T|\{\tau^* > T\}]$. To solve problem (12), we use the same scheme of the simulation experiment as in the case of the isolated MIC.

**Results of the Simulation Experiment**

Fig.3 shows the dynamics of the flows $r^t$ and $f^t$, obtained as a result of solving the problem (12) with the value of the rate of return of the center’s own funds $g = 0.1$ (on the left) and $g = 0.25$ (on the right) and the value of $d$ equal to 0.2. The required reduced income of the Center from participation in the financing of the MIC $G_0$ is chosen to be zero.

In the case, if $g < d$ MIC participants, it is profitable to make an initial replenishment of the fund, attracting loans from the Center (loans) with their subsequent return by raising the level of premiums in the last years of the fund’s work. Thus, the Center acts as a credit institution.

In the case, if $g > d$ MIC, the opposite strategy turns out to be more effective: to channel part of the premium flow to the Center, and at the expense of subsequent refunds, reduce the level of insurance premium in recent years. This model can be interpreted as an investment of insurance reserves.

In the left part of Fig.4 there is a graph characterizing the effect for policyholders of interaction with the Center: if there is a difference in rates $g$ and $d$. Regardless of its absolute value, the attraction of additional funds substantially removes the cost of insurance for the members of the MIC in comparison with its analog in its isolated work.

In the right part of Fig.4 a graph showing the effect of the level of the required income of the Center $G_0$ on the cost of insurance for a fixed difference in rates $d - g = 0.1$ is shown. The area to the left of the ordinate axis characterizes the cost of insurance with preferential (non-commercial) forms of support for the MIC by Center. In the example, when the Center is ready to incur a loss in the amount of one expected payment for the MIC portfolio, provided the distribution of financial flows is optimal, the cost of insurance is reduced by more than 1.5 times. Such interaction can be interpreted as preferential lending or subsidization by the Center for MIC activities.

The area to the right of the ordinate axis, on the other hand, reflects the results of the Center’s interaction with the MIC on commercial terms. At the used values of the parameters, the Center can extract the reduced income in the amount of up to 2.7 volumes of expected payments on the
insurance portfolio of the MIC, provided that the cost of insurance for the members of the MIC will be less than its equivalent corresponding to the isolated work of the company.

**Conclusion**

Mutual Insurance Companies (MIC) are an important part of the global insurance market, and in some of its segments, it is possible to provide more effective insurance coverage of participants, in comparison with commercial insurance companies. However, at the initial stage of their activity, the MIC is characterized by relatively low financial stability in view of the insufficiency of its own capital and the complexity of increasing its fund at the expense of the participants. In such conditions, the increase in the attractiveness of a MIC as a type of insurance protection is
associated with the optimization of strategies for the formation of an insurance fund.

The solution of the problem of optimizing the management of the process of accumulation of the insurance fund for MIC under the conditions of a significant number of variables that are considered to be random, with known laws of their distribution, for the sake of simplifying the calculations, is expedient to obtain on the basis of simulation methods.

The results of the simulation experiments show that the least expensive from the point of view of the policyholders is the formation of a MIC fund at the expense of its own funds. It is a large entrance fee that is several times higher than the actuarially fair level of the premium, followed by an annual replenishment of the fund with small payments.

The attraction of an external source of financing (the Center) to non-commercial and to certain limits on commercial terms allows the MIC to reduce the cost of insurance for its members in comparison with its counterpart in the isolated work of the society. The obtained results as a whole testify to the existence of real possibilities for increasing the financial stability of the MIC at the initial stage of activity due to the rationalization of the regimes of filling its fund.

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References