Construction and Robustness of Interdependent Networks via Time Series and Visibility Graph

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Abstract. In this paper, based on time series, visibility graph method and positive correlated coupling, a novel algorithm of constructing interdependent network is proposed. Using this new algorithm, an interdependence network with two subnets (periodic sequence subnet and Conway sequence subnet) is generated. Afterwards, statistical characteristics of this network are analyzed. Finally, with malicious attack, the robustness of the constructed interdependent network is discussed. We find that if the attacked subnet is the periodic subnet, the interdependent network has good robustness when attacking the nodes with larger degrees, and it has poor robustness when attacking the nodes with smaller degrees. If the attacked subnet is Conway sequence subnet, regardless of the attacked nodes with larger or smaller degrees, the network has good robustness.

Introduction

For the complex networks, so far, most researches tend to analyze complex networks with single component. However, with the increasing development of science and technology, the interaction and inter-dependence between various complex systems will become stronger and stronger. For example, the human body can be viewed as an interdependent system consisting of the cardiovascular systems, the respiratory systems, the brain neuron systems, and the nervous systems. Banks, insurance companies, and commercial companies compose an interdependent complex system. Therefore, we should pay more attention to complex systems composed of multiple networks with inter-dependent links, that is, interdependent network.

Compared with single complex networks, the history of using complex network tools to investigate interdependent systems is very short. The current research mainly focuses on the establishment of the interdependent networks and robustness analysis. The literatures [1-2] pointed out that due to the interdependence, failures in one infrastructure were spread to other infrastructures, and lead to extensive spread of damages. Although interdependence of a system has been studied for more than a decade, most of them are empirical studies. In 2010, for the first time, Buldyrev et al. [3] used complex network theory to propose a network model with the same number of nodes which are linked interdependently in two directions. Meanwhile, they analyzed its robustness and proposed the concept of cascade failure. From then on, it has opened the door to the investigation of interdependent networks from the perspective of complex networks. Afterwards, some results of model construction and robustness analysis of interdependent networks have appeared successively [4-9]. However, these researches merely analyze the backbone structure between sub-networks without considering the internal structure of the sub-network.

As for the construction of complex network, the time series-based visibility method has attracted much attention because of its simplicity. In 2008, for the first time, Lacasa [10] proposed the visibility graph algorithm (VG) from the time series to the network, and illustrated that this algorithm can convert periodic time series into regular networks, random sequences into random graph networks, and chaotic fractal sequences into small-world and scale-free networks. In 2009, Luque [11] put forward a horizontal visibility graphs algorithm (HVG), which is simpler and less complex than the VG method. In the literature [12], Zhou et al discussed an improved viewable algorithm—limited penetrable visibility graph (LPVG). In the same year, Nunez [13] added a noise filter to the HVG and
proposed a filtered Horizontal visibility graphs (fHVG). Ahadpour et al.\textsuperscript{[14]} obtained a viewable method based on VG algorithm to build a complex network for 0-1, namely sequence—binary visibility graph (BVG). Gao et al.\textsuperscript{[15]} discussed a novel multiscale limited penetrable horizontal visibility graph. Luque and Lacasa\textsuperscript{[16]} showed that, under suitable conditions, there exist a bijection between the adjacency matrix of an HVG and its degree sequence, and gave an explicit construction of such bijection.

Based on the statement mentioned above, we investigate the construction and robustness of interdependent networks via time series and visibility graph in this paper. In order to obtain two sub-networks, firstly, based on the visibility graph method, two subnets of the interdependent network are constructed with periodic time series and Conway fractal time series, separately. Secondly, each node of these two sub-networks is one-to-one positive correlated inter-coupled. Thirdly, we analyze the statistical characteristics of this generated network. Finally, the robustness of the interdependent network is discussed.

**Preliminaries**

In order to better illustrate the algorithm proposed in this paper, the visibility graphs, Conway fractal time series will be briefly introduced in this section.

**Visibility Graph**

In 2008, Lacasa\textsuperscript{[16]} proposed a visibility graph (VG) method that maps time series into complex networks.

The idea of the algorithm is shown in Figure 1. The first 20 data of the periodic time series are represented by 20 straight bars in Figure 1 (a). The height of the bar is the data value. If the tops of the two bars are visible for each other, the two points are considered to have a link in the network. Figure 1 (b) is a network developed by the VG algorithm for the periodic time series. The nodes in the figure correspond to the data in the time series. The connections between the edges in the network and the bars are in one-to-one correspondence. It is noted that the nodes cannot have edges with themselves, and the visible lines between the two bars cannot cross other bars. The principle for the visibility method is in the following.

For any two data \((x_a, y_a)\) and \((x_b, y_b)\) in the time series, if they are visible, then for any \((x_c, y_c)\), where, satisfies

\[
y_c < y_b + (y_a - y_b)(t_b - t_c)/(t_b - t_a)
\]  

**Conway Fractal Time Series**

The recurrence relation of the Conway fractal time series is as follows.

\[
\begin{align*}
a(n) &= a(a(n-1)) + a(n-a(n-1)) \\
a(1) &= a(2) = 1
\end{align*}
\]  

Figure 2 is the Conway fractal time series consisting of 1000 sequence nodes.
Construction of Interdependent Networks

In this section, based on the visibility graph method and time series, and degree correlation, a novel algorithm for establishing an interdependent network is proposed. The network includes two subnets which are obtained from a periodic time series and a Conway fractal time series. The model evaluation for the proposed network is followed.

Algorithm Description

The algorithm can be described by the following three steps.

Step 1: Given two equal length time series, \( X = \{X_i\}_{i=1,2,..,n} \) and \( Y = \{Y_i\}_{i=1,2,..,n} \), where each sequence includes \( n \) samples.

Step 2: Construct the internal structure of the two subnets for the interdependent network. According to VG algorithm, the complex networks A and B with \( n \) nodes are brought forward with the two time series \( X \) and \( Y \), respectively.

Step 3: Construct the backbone structure between the two subnets A and B. Note that the subnets A and B are inter-connected by one-to-one correspondence. That is to say that the two subnets are completely dependent on each other.

Remark 1: In this paper, the principle of inter-connection between subnets is positive correlated couplings. That is, the node with the largest degree in one subnet is connected to the node with the greatest degree in the other subnet, and so on.

Construction and Evaluation of Subnets

Based on step1 and step2, in this section, two subnets are constructed by the periodic time series and the Conway fractal time series, respectively. At the same time, the statistical features of each subnet are analyzed.

The Periodic Time Series. Let the periodic time series be \( Y = \sin(5x) \), the sampling interval \( \Delta x = 0.01 \) and the sequence length \( n=1000 \). According to step1 and step2, Figure 3 displays the constructed subnet which has 10 large-scale clusters. With Matlab, the average degree of the sub-network is 118.283. It is large. The average path length of the network is 7.723, which implies the small-world characteristics of the network. The clustering coefficient of the network is 0.626 and its diameter is 40. For Figure 4, because the sinusoidal signal is a single-cycle time series, the degree distribution displays only one sharp peak.

Conway Fractal Time Series. Let the time series be the Conway fractal time series, as shown in Figure 2. The sub-network constructed by the Conway fractal time series is shown in Figure 5. The average degree of the network is 27.932. The average path length of the network is 11.361, which is longer than the network proposed by the periodic time series. The clustering coefficient of the network is 0.593 and its diameter is 37. At the same time, Fig.6 displays that the degree distribution of the sub-network.

Figure 4. The degree distribution of subnet.

Figure 5. Sub-network (Conway fractal time series).

Figure 6. The degree distribution of subnet.
Construction and Evaluation of Interdependent Network

According to the two subnets constructed in the previous section, in this section, using step 3 of the algorithm, a novel positive correlated coupled interdependent network is obtained, and the statistical characteristics of the network are analyzed and evaluated.

If the sub-network generated are connected with positive correlated relationships, we can obtain an interdependent network (see Figure 7).

Figure 7. An interdependent network.

Figure 8. The degree distribution of the interdependent network.

Figure 9. The interdependent network after attacking.

Figure 8 shows the degree distribution of the interdependent networks. The average degree of the generated network is 74.108, which is almost the arithmetic mean of the original two sub-networks degrees. This is because the backbone structure of the network is connected by one to one correspondence, and the degree of each node is almost unchanged. This can be observable by comparing the degree distribution in Fig.8 with Fig.4 and Figure 6. The average path length of the network is 3.946, which is smaller than the average path length within the two subnets. The clustering coefficient of the network is 0.468, which is smaller than in the two subnets. The network diameter is 9.

Robustness of the Interdependent Network

In this section, the robustness of the constructed interdependent network is investigated. As we all know that when a node is attacked, the node fails, and the connected edge of the subnet and the dependent edge between the sub-networks will lose their functions. After that, cascade failure will follow. In this paper, we consider malicious attacks. Above all, the nodes of the attacked subnet are sorted, and then some nodes with large degree or small degree in the attacked subnet are removed. Simultaneously, we discuss the cascade failure which will influence the robustness of the interdependent network. With the different subnet and nodes chosen in the network when attacking, the robustness of the network will change accordingly.

Malicious Attack in the Periodic Time Series Subnet

If the attack is performed on the 100 nodes with larger degree of the periodic time series subnet, the interdependence network and degree distribution are shown in Figure 9 and Figure 10, respectively. Now, the average degree of the attacked network is 62.236. Because the nodes with larger degree are deleted, the average degree is smaller than the original network. The average path length is 4.154, the clustering coefficient of the network is 0.429 and the network diameter is 7. These statistics are almost no different from the original network. Compared with Figure 8, most of the degree distributions are not changed. They imply that though the attacked nodes have larger degree, the network connectivity is almost very well. Therefore, the network has good robustness against malicious attacking nodes with larger degree.

If 100 nodes with smaller degree of the periodic time series subnet are attacked, the result is shown in Fig.11. At this time, the degree of the network is 72.247. Since the nodes with smaller degree are deleted, the difference of degree with the original network is not great. The network diameter is 54, which is much larger than the original network’s, which means that the network is damaged seriously. The clustering coefficient of the network is 0.233, which is smaller than the
original network. It tells us that the network becomes more dispersed. At the same time, the average path length is 11.095, which is also much larger than the original network.

Compared with Figure 8, there is no significant change in Figure 12. This is because the attacked nodes have smaller degree. But, the network clustering becomes smaller, the diameter larger, the average path length larger, the network connectivity becomes worse. So, the network has bad robustness against malicious attacking the nodes with smaller degree of periodic time series sub-network.

Malicious Attack in the Conway Time Series Subnet

Similarly, if the subnet constructed by the Conway sequence is attacked. First, the 100 nodes with larger degree of the subnet are attacked, and the network graph and degree distribution after attacking are analyzed, as shown in Figure 13 and Figure 14.

At this time, the degree of the network is 67.698, the diameter is 9, the clustering coefficient is 0.468 and the average path length is 4.358. Compared to the original network, all of these statistics change a little. In general, the structure of the network is similar to the original network’s, which means that the network has good robustness against malicious attacking.

When we attack 100 nodes with smaller degree nodes of the Conway subnet, Figure 15 illustrates the attacked network. After attacking, the degree of the network is changed to 74.708, the network diameter is 7, the clustering coefficient is 0.480 and the average path length is 4.092. Compared with the statistical results in the original network, the structure is similar to the original network’s, and the connectivity is less disruptive, which implies that the robustness is good when attacking the nodes with smaller degree of the Conway sub-network.

Conclusion

In this paper, the problems of the construction and the robustness of interdependent networks are discussed. First of all, drawing upon time series and the visibility graph method, we construct two subnets with the periodic time series and Conway fractal time series, respectively. Then, an interdependent network is developed when the two subnets are positive correlated inter-coupled. Afterwards, we analyze the statistical features of the generated network. Finally, simulations
explore the robustness of the constructed interdependent network with periodic and Conway time series subnets when malicious attacking. Generally, if the attacked subnet is the periodic series subnet, the interdependent network has poor robustness when attacking the nodes with smaller degree, but it has better robustness when attacking the nodes with larger degree. If the attacked subnet is the Conway series subnet, the network has good robustness no matter whether the attack node has large or small degree. These results will provide insights into understanding the construction and robustness of interdependent networks, and may potentially be applied to the real networks, such as interdependent infrastructure.

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References


