Performance Analysis of Mimo-Scma Systems for 5G Communications
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ABSTRACT

Recently, sparse code multiple access (SCMA) is proposed as one of the important candidacies for the fifth generation (5G) radio access technique. SCMA is a non-orthogonal multiple access (NOMA) technique which allows multiple users share the same radio resource and thus increase spectral efficiency largely. In SCMA, a set of binary coded bits is mapped directly into a complex vector by use of a predefined multi-dimensional codebook. The sparsity of the codewords makes the near-optimal detection feasible through iterative message passing algorithm (MPA). The pursuit of higher spectral efficiency motivates us to study the performance of multiple-input-multiple-output (MIMO) SCMA system in this paper. Simulation results show that compared with the conventional MIMO-OFDM system, MIMO-SCMA can provide a gain up to 50% in data throughput with the aid of MPA technique.

KEYWORDS: Codebook, MIMO, MPA, SCMA, 5G

1 INTRODUCTION

The exponential growth of wireless data services driven by mobile Internet and smart devices has raised new challenges to the current established fourth generation (4G) wireless communication systems. In order to provide greatly enhanced capacity, energy efficiency, spectral efficiency, cost efficiency, mobility, connection density, etc., with much reduced end-to-end latency, the investigation of the fifth generation (5G) wireless communication networks has been triggered. Around 2020, the new 5G mobile networks are expected to be deployed. In 5G, a lot of new network architectures and technologies, such as heterogeneous ultra-dense networks, massive multiple-input-multiple-output (MIMO), millimeter wave communications, full duplex, etc., will be investigated and applied [1]-[3].

In conventional wireless communication systems, such as 4G, the orthogonal resource, such as different subcarriers, are allocated to different users to make sure that they can access the system without interfering each other [4]. Orthogonal multiple access (OMA) is a reasonable choice for achieving good system performance in packet-domain services with a simplified receiver design. However, in order to increase spectral efficiency, more users are expected to be accommodated into the same resource which leads to a new technique, named
non-orthogonal multiple access (NOMA) [5] [6]. In NOMA, more advanced receiver technique should be employed to mitigate the interferences generated by the co-existing users. Recently, sparse code multiple access (SCMA) is proposed as one of the candidates for 5G radio access technique [7]-[10]. SCMA is a multi-dimensional codebook-based NOMA technique. It is developed based on low density signature (LDS) which is a special approach of CDMA sequence design with a few numbers of nonzero elements within a large signature[11][12]. In [8], the systematic approach to design SCMA codebooks is proposed. The multi-user SCMA technique is introduced in [9] for downlink wireless access. In [10], the energy efficiency is analyzed and the low complexity receiver design is proposed. Through the computer simulation and the measurement from prototyping system, SCMA is regarded as an energy-efficient approach for 5G wireless communication systems. On the other hand, MIMO is considered as one of the most promising breakthrough technology in wireless communications for its capability of providing high data rates (spatial multiplexing) and/or enhanced reliability (diversity) [13][14]. So, it is reasonable to use the combination of MIMO and SCMA to further increase spectral efficiency. In this paper, we introduce the MIMO-SCMA system and analyze its performance through computer simulations.

The rest of the paper is organized as follows. Section II introduces the basic idea of the SCMA technique and describes the sparse structure of the codewords and codebooks in SCMA. In Section III, we propose the signal model of MIMO-SCMA system and introduce the most used ZF/MMSE equalization technique. The sparsity of the codewords makes the near-optimal detection feasible through iterative message passing algorithm (MPA) which is described in Section IV. Numerical results are reported in Section V. The paper finally concludes in Section VI.

2 SCMA ENCODER

Generally, an SCMA encoder can be viewed as an extended modulator, since it maps a set of binary coded bits directly into a complex vector [10]. This complex vector is a codeword selected from a codebook and transmitted directly on the orthogonal resources. Thus, in order to transmit \( \log_2(M) \) coded bits on K orthogonal resources, a K-dimensional complex codebook of size M needs to be predefined. In SCMA, the K-dimensional complex codewords of the codebook are all sparse vectors, since each one of them only has \( N < K \) non-zero entries and the other entries are all zeros. Compared with the conventional modulation procedure, the codebook size M can be viewed as the number of alphabets and each codeword corresponds to a K-dimensional constellation points.

An SCMA encoder contains J separate layers, on which J independent data streams are transmitted [7]. These layers may be employed by one user or belong
to different users according to the system deployment. On the j-th layer where \(0 \leq j \leq J - 1\), a K-dimensional complex codebook of size \(M_j\) is defined to transmit \(\log_2 (M_j)\) coded bits on K orthogonal resources. The K-dimensional complex codewords are sparse vectors with \(N_j < K\) non-zero entries. All the codewords in the codebook contain 0 in the same \(K - N_j\) dimensions. If we assume that all layers have the same codebook size and non-zero entries, i.e. \(M_j = M, N_j = N\), for \(0 \leq j \leq J - 1\), the transmitted symbols in SCMA systems can be expressed by

\[
X = \sum_{j=0}^{J-1} \sqrt{P_j} x_j
\]

(1)

where \(P_j\) is the power allocated to the j-th layer and 
\(X = [X(0), X(1), ..., X(K-1)]^T\). \(x_j = [x_j(0), x_j(1), ..., x_j(K-1)]^T\) is the codeword on the j-th layer with \(\|x_j\| = K\). \(x_j\) is selected from the j-th codebook according to the input information bits on the j-th layer. Note that due to the sparsity property of SCMA encoder, some entries in \(x_j\) are 0.

The whole structure of SCMA symbol can be illustrated by an indicator matrix \(F = [f_0, f_1, ..., f_{J-1}]\), where \(f_j\) is a K × 1 indicator vector. The \(k\)-th element of \(f_j\) is denoted by \(f_{k,j}\) and is given by [11]

\[
f_{k,j} = \begin{cases} 
0, & \text{then } x_j(k) = 0; k = 0,1, ..., K-1. \\
1, & \text{then } x_j(k) \neq 0; k = 0,1, ..., K-1.
\end{cases}
\]

(2)

the position of 1’s in \(f_j\) denotes the position of non-zero entries in the codeword of layer k. Let \(\xi_k\) and \(\zeta_k\) be the set of 1’s position in the k-th row and in the j-th column in \(F\), respectively, then we can modify (1) as

\[
X(k) = \sum_{j \in \xi_k} \sqrt{P_j} x_j(k) = x^{[k]}
\]

(3)

where \(x^{[k]}\) denotes the vector stacking the signals transmitted by every layer participating in orthogonal resource \(k\).

The principle of SCMA encoder can be illustrated by an example shown in Figure 1 for the convenience of readers. If the input bits on the j-th layer is \(\{1, 0\}\), a codeword of \(x_j = [0, a3, 0, b3]^T\) is chosen from the j-th codebook. Finally,
the J codewords are added and the output signal vector $\mathbf{X}$ is transmitted directly on K orthogonal resources. Since there are J layers transmitted on K resources, the overloading factor of SCMA system is $J / K$.

![Codebook in SCMA](image)

Figure 1. Codebook in SCMA.

3 MIMO-SCMA SYSTEM MODEL

We consider a MIMO system with M transmit antennas and M receive antennas. On the m-th antenna, a signal vector $\mathbf{X}_m = [X_m(0), X_m(1), ..., X_m(K-1)]$ is transmitted. $\mathbf{X}_m$ is the output of SCMA encoder and contains the codewords from J layers with the same codebook size and non-zero entries. In this paper, we assume that all the J layers and the M signal vectors are employed by one user which corresponds to a signal user MIMO-SCMA case. If the subcarrier of OFDM is used as the orthogonal resource, the channel matrix can be written as

$$
\mathbf{H}(k) = \begin{bmatrix}
H_{1,1}(k) & H_{1,2}(k) & \cdots & H_{1,M}(k) \\
H_{2,1}(k) & H_{2,2}(k) & \cdots & H_{2,M}(k) \\
\vdots & \vdots & \ddots & \vdots \\
H_{M,1}(k) & H_{M,2}(k) & \cdots & H_{M,M}(k)
\end{bmatrix}
$$

(4)
where $H_{l,m}(k)$ denotes the channel frequency response (CFR) on the $k$-th subcarrier between the $m$-th transmit antenna and the $l$-th receive antenna.

Under the assumption that the synchronization is perfect and the channel is constant during the transmission of one OFDM symbol, the received signal on the $k$-th subcarrier can be expressed by

$$Y(k) = \sqrt{\frac{E_s}{M}}H(k)X(k) + V(k)$$

where $E_s$ is the total transmit power, $X(k) = [X_0(k), X_1(k), ..., X_{M-1}(k)]^T$ denotes the transmit signal and $V(k) = [V_0(k), V_1(k), ..., V_{M-1}(k)]^T$ is the additive white Gaussian noise (AWGN) with zero mean and covariance of $\sigma_n^2I$, where $I$ indicates the identity matrix. In this paper, we assume that the complex-valued channel matrix $H(k)$ has i.i.d. Gaussian entries with zero mean and unit variance and all the channel matrices on the K subcarriers are identical, i.e. $H(0) = H(1) = ... = H(K)$. For the sake of notation simplification, we drop the index $k$ from $H(k)$ in the following analysis. Figure 2 shows the block diagram of the described MIMO-SCMA system.

Assume the channel is unknown to the transmitter but perfectly known to the receiver. In this case, uniform power allocation at transmitter is preferred. If we set all the powers of different layers on the different antennas to be identical, i.e. $P_0 = P_1 = ... = P_{j-1} = P$, the transmit power of each antenna is equal to $JP$ and the total power is thus $E_s = MJP$.

![Figure 2. MIMO-SCMA system.](image)

### 3.1 ZF Equalization

At receiver, although the maximum likelihood (ML) detection achieves the optimal performances, it suffers from the exponentially increasing computational
complexity. Besides the ML criterion, the zero forcing (ZF) and minimum mean square error (MMSE) detections are linear equalization-based methods. They are low-complexity and simple to implement with degraded performances. In ZF detection, the received signal is multiplied by an equalization matrix $G_{ZF}$, which is the Moore-Penrose pseudo-inverse of $H$,

$$G_{ZF} = (H^H H)^{-1} H^H$$  \hspace{1cm} (6)$$

where $(\cdot)^{-1}$ and $(\cdot)^H$ denote inverse matrix and hermitian transpose, respectively. After ZF equalization, we obtain

$$Z(k) = G_{ZF} Y(k) = (H^H H)^{-1} H^H Y(k)$$ \hspace{1cm} (7)$$

where $Z(k)=[Z_0(K), Z_1(K), \ldots, Z_{K-1}(K)]^T$ is the estimate of the transmitted symbol vector. According to [15] [16], the signal-to-noise ratio (SNR) of the $k$-th transmitted signal can be expressed by

$$\text{SNR}_m = \frac{|g_m h_k|^2 E_i}{M \sigma_n^2 \|g_m\|^2 + \sum_{j=1}^{m-1} |g_m h_j|^2 E_i} = \frac{E_i}{M \sigma_n^2 \left[H^H H\right]_{m,m}^{-1}}$$ \hspace{1cm} (8)$$

where $g_m$ denotes the $m$-th row of the equalization matrix $G_{ZF}$ and $h_k$ represents the $m$-th column vector of the channel matrix $H$.

### 3.2 MMSE Equalization

The concept that MMSE detection uses is to minimize the mean square error $\mathbb{E}\left[\|GY(k) - X(k)\|^2\right]$, where $G$ denotes the equalization matrix. The MMSE detection considers the noise variance and reduces the noise enhancement by using the minimum mean square error equalization matrix obtained from

$$G_{MMSE} = \left(H^H H + \frac{M \sigma_n^2}{E_i} I\right)^{-1} H^H$$ \hspace{1cm} (9)$$

The transmitted symbol vector estimate of the MMSE detection is written as

$$Z(k) = G_{MMSE} Y(k) = \left(H^H H + \frac{M \sigma_n^2}{E_i} I\right)^{-1} H^H Y(k).$$ \hspace{1cm} (10)$$
After MMSE equalization, the received signal is decoupled into $M$ substreams with output SNR

$$\text{SNR}_m = \frac{E_s}{\sigma^2 M} - 1, 0 \leq m \leq M - 1$$ (11)

4 MPA DECODER

From $K$ transmitted symbol vector estimates, we obtain $M$ SCMA symbol estimates. They are $Z_m = [Z_m(0), Z_m(1), \ldots, Z_m(K-1)]$, $0 \leq m \leq M - 1$. These SCMA symbol estimates are then fed into the following independent MPA decoder to retrieve the input binary bits. Without loss of generality and for notational brevity, we drop the index $m$ from $Z_m$ in the following analysis.

Given the $Z_m$, the joint maximum-a-posteriori (MAP) detection of $X_m$ can be written by [10] [11]

$$\hat{X} = \arg\max_{X \in \chi'} P(X|Z)$$ (12)

where $\chi'$ represents the codebook of the $j$-th layer. MAP detection requires a globe search over the joint codebook space of $J$ layers $\chi^0 \times \chi^1 \times \ldots \times \chi^{J-1}$ to minimize the probability that all layers are detected correctly. Due to the sparsity property of SCMA codewords, the MAP detector problem can be translated into marginalize product of functions (MPF) problem much more efficiently.

According to [11], the codeword $x_j$ can be estimated by

$$\hat{x}_j = \arg\max_{w \in \chi'} \sum_{X \in \chi_0 \times \chi'_0 \times \ldots \times \chi'_{J-1}} P(X) \prod_{k \in j} p(Z(k)|x_j^{[k]})$$ (13)

where $w$ is the selected codeword and $P(X) = \prod_{j=0}^{J-1} P(x_j)$.

The MPF problem can be solved by message passing algorithm (MPA) proposed in [11] onto the underlying factor-graph $\Omega(U,C)$. Factor-graph is the bipartite graph representation of the factorization defined in (13). In this factor graph, the transmitted symbols $x_j$, $j = 0, 1, \ldots, J - 1$ and the observation $Z(k)$, $k = 0, 1, \ldots, K - 1$ are denoted as variable nodes $u \in U$ and function nodes $c \in C$,
respectively. At the function node $c_k$, the local observation $Z(k)$ is made and given by

$$p(Z(k)|x^{[k]}) = \frac{1}{\sqrt{2\pi\sigma_n}} \exp\left(-\frac{1}{2\sigma_n^2}\|Z(k) - x^{[k]}\|^2\right)$$  \hspace{1cm} (14)

Figure 3 shows an example of factor-graph representation for a SCMA encoder with $K = 4$, $N = 2$ and $J = 6$. The indicator matrix is assumed to be

$$F = \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 1
\end{bmatrix}$$  \hspace{1cm} (15)

Figure 3. Factor-graph representation.

5 SIMULATION AND RESULTS

Several computer simulations are carried out in this section to evaluate the performance of a MIMO-SCMA system with two transmit antennas and two receive antennas. The channel is assumed to be static over one SCMA symbol duration and variable independently on a symbol-by-symbol basis. We consider a SCMA encoder with $J = 6$, $K = 4$ and $N = 2$. The codebooks proposed by Huawei Technologies Co., Ltd are adopted in simulations. They are:

- $[0 \ 0 \ 0 \ 0; -0.1815 - 0.1318j - 0.6351 - 0.4615j 0.6351 + 0.4615j 0.1815 + 0.1318j]$;
- $[0.7851 - 0.2243j 0.2243 - 0.7851]$;
- $[-0.6351 + 0.4615j 0.1392 - 0.495 - 0.495j]$;
0.1759j 0.4873 − 0.6156j − 0.4873 + 0.6156j − 0.1392 + 0.1759j; 0 0 0 0; 0 0 0 0] ; [0 0 0 0; 0 0 0 0; 0.7851 − 0.2243 0.2243 − 0.7851; −0.0055 − 0.2242j − 0.0193 − 0.7848j 0.0193 + 0.7848j 0.0055 + 0.2242j; [−0.0055 − 0.2242j − 0.0193 − 0.7848j 0.0193 + 0.7848j 0.0055 + 0.2242j; 0 0 0 0; 0 0 0 0; −0.6351 + 0.4615j 0.1815 − 0.1318j − 0.1815 + 0.1318j 0.6351 − 0.4615j; [0 0 0 0; 0.7851 − 0.2243 0.2243 − 0.7851; 0.1392 − 0.1759j 0.4873 − 0.6156j − 0.4873 + 0.6156j − 0.1392 + 0.1759j; 0 0 0 0]. For the purpose of comparison with the OMA systems, a 2 × 2 MIMO-OFDM with QPSK modulation is also simulated. In simulation, ideal channel estimation is assumed and no channel coding is considered. The number of iteration cycles in MPA is set to 5.

Figure 4 shows the performance comparisons as a function of SNR in term of achievable data rate. Achievable data rate is defined as the averaged successfully received bits on every four subcarriers. From Figure 4, we can see that up to 50% gain in data throughput can be achieved by the MIMO-SCMA system with the aid of MPA technique, when compared with MIMO-OFDM system.

![Figure 4. Achievable data rate.](image)

### 6 CONCLUSIONS

SCMA is a new multiple-access technique for it can takes the advantages of CDMA and low density signature (LDS) as well as some potential gains of multi-dimensional codebook design. The sparsity of the codewords makes the
near-optimal detection feasible through iterative MPA. In this paper, the basic principle of the SCMA encoder and decoder are introduced and the performance of MIMO-SCMA system is analyzed through computer simulations. Compared with MIMO-OFDM system, MIMO-SCMA can provide a gain up to 50% in data throughput with the aid of MPA technique which demonstrates that MIMO-SCMA is a promising radio access technique for the upcoming 5G communication networks.

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