An Equilibrium Model of Diagnostic Information Transmission Network

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ABSTRACT

Diabetes is one of the most typical chronic. There are more undiagnosed diabetics and pre-diabetics (UD & PD) than diagnosed diabetes. In this study, we applied a three-tier diagnostic information transmission network equilibrium model to support UD & PD in making an informed choice that is in line with their personal values. Considering the characteristics of diabetes diagnosis information, this paper designed the network as follows: first tier, health professionals; second tier, diabetes educators; third tier, UD & PD. An equilibrium model is established up on the network which is sufficiently general to handle the medical decision-makers. Such a model is sufficiently general to handle many medical decision-makers and their independent behaviors. The structure of the diagnosis information transmission network is identified and equilibrium conditions are derived. A finite-dimensional variational inequality formulation is established. Qualitative properties of the equilibrium model and numerical examples are given.

Keywords: Information transmission network; Equilibrium model; Variational inequality formulation; Equilibrium point; Decision making

INTRODUCTION

The World Health Organization (WHO) issued its first global report on diabetes on April 6, 2016[1]. China has the world's largest number of diabetic people[1]. Type 2 diabetes is the most prevalent form of diabetes, which accounts for at least 90% of all cases of diabetes[2, 3]. Since there is no active detection, diabetic patients are likely to developing into diabetics in the absence of control. And some may be undiagnosed diabetics without knowing it because they have no serious symptoms and never seek for medical care[4]. Although type 2 diabetes is difficult to cure, progression can be delayed and prevented. There is a large amount of evidence that good self-management, such as a healthy diet and moderate physical activity, can help prevent the development of type 2 diabetes[5-7]. As a consequence, extensive research on risk factors associated with type 2 diabetes has been conducted[8-10]. In 1952, Wardrop
proposed two principles on traffic equilibrium, namely, user equilibrium and system optimality. Mihalák and Schlegel (2012) introduce and study the concept of an asymmetric swap-equilibrium for network creation games and introduce a node-weighted version[11]. Brekke et al (2017) draw on economics, decision theory and operations research, offered a simple guide on how to transform a deterministic energy market equilibrium model - where several agents simultaneously make decisions - into a stochastic equilibrium model[12]. Zhou & Cao (2014) studied the equilibrium structure of two competing supply chains and find that that both price and displayed-quantity competition intensities influence significantly the equilibrium structure[13]. Variational inequality was first introduced as a mathematics subject in 1966 by Hartman and Stampscchia[14]. Since then variational inequalities have been extended and generalized in several directions, especially in equilibrium model.

This study was motivated by the need of lightening diabetes burden in China. Reducing undiagnosed diabetes and reversing pre-diabetes are something that worth doing for this purpose. In this study, we developed a Diagnostic Information Transmission Network (DITN) to study the promotion of diabetes screening and prevention through the alert given by diabetes diagnostic information. This paper intends to start from the characteristics of diabetes diagnostic information and establish a balanced model based on the diagnostic information flow. Thus, the equilibrium can be reached based on a certain amount of information then the need of most effective information transmission is meted, diabetes burden is expected to be reduced.

Our main contribution is adopting the equilibrium model to analyses the diagnostic information transmission, which is depicted by the three-tier diabetes DITN and solved by the variational inequality. For the studied problem, standing on the perspective of UD & PD, a balanced information delivery is solved by the proposed model. Thus, our model is proven to be very useful for UD & PD’s decision making and guiding the related health workers to carry out the propaganda and education of the disease, lightening the burden of people's diseases. The proposed methodology is easy to implement for any similar situation in practice.

**MODEL DEVELOPMENT FOR THE DITN**

**Background of the Model**

The DITN we developed in this paper consists of three groups of participants: health professionals, diabetes educators and UD & PD. Based on the DITN, the equilibrium model is then constructed along with the variational inequality formulation of the governing equilibrium conditions. Some mathematical notations are provided in Appendix A.

Through information sharing, which we call transmission, the amount of information per unit may expand or shrink relative to the initial amount. To better depict the information transmission in the DITN, we need the sum of the total information at each tier. Therefore, some equivalence relations are needed before the model is developed. The equivalence relations are given as follows:
\[ d_i^0 = \sum_{j=1}^{n} d_{ij}^0 ; \quad d_j^0 = \sum_{i=1}^{m} d_{ij}^0 ; \quad d_j = \sum_{k=1}^{p} d_{jk} \]

make \( t_j = \frac{d_j^0}{d_j} \) and \[ \begin{cases} d_i = t_j d_i^0 \\ d_{ij} = t_j d_{ij}^0 \end{cases} \] (1)

Problem description of DITN

![Network structure of the transmission network of diabetes diagnostic information.](image)

(i) First tier: Includes \( m \) health professionals, which refers to individuals who work to help to maintain the health of their clients.

(ii) Second tier: Includes \( n \) diabetes educators who spread knowledge about diabetes. They obtain the information about diabetes directly or indirectly from the health professionals. They help patients obtain better knowledge of the disease.

(iii) Third tier: Contains \( p \) people who must decide whether to go for a diagnosis or not according to the information they receive from the diabetes educators—undiagnosed diabetic and pre-diabetic people (UD & PD).

Through Figure 1., this paper sets the equilibrium conditions governing the DITN with complete information to be represented by the solution to the variational inequality problem. Through the application of the variational inequality method, a quantitative optimal information transmission is given to the DITN (Article Framework see Appendix B).

THE BEHAVIOR OF THE DITN

The Behavior of Health Professionals

We use \( d_i^0 (d_i^0 \geq 0) \) to denote the diagnostic information outflow from the health professional \( i \). The health professionals can be nurses, pharmacists, doctors, etc.

We group the information outputs of all health professionals into a column vector \( d^0 \in K^m_\uparrow \). We assume that each health professional \( i \) is faced with an output
loss of information $l_i$, which depends, in general, on the entire vector of information outputs, that is,

$$l_i = l_i(d) \quad \forall i \in Z^+$$

(2)

We denote the information transmitted from health professional $i$ to $j$ as $d_{ij}^0$. The error and loss of information transmission between $i$ and $j$ is denoted as $e_{ij}$, which may be caused by understanding bias, imperfect communication tools, etc. We group the information transmission between the upper two tiers into the $m \times n$-dimensional column vector $D^1$. Therefore, the error and loss of information between $i$ and $j$ can be given by the following:

$$e_{ij} = e_{ij}(d_{ij}) \quad \forall i \in Z^+, j \in Z^+$$

(3)

To help fix ideas, and to better describe the equilibrium structure of the network between health professionals and diabetes educators, we depict the health professionals and diabetes educators as nodes and the transactions between health professional $i$ and diabetes educator $j$ as links (Figure 2.)

The quantity of the output information by health professional $i$ must satisfy the equations as follows:

$$d_i^0 = \sum_{j=1}^{n} d_{ij}^0 d_i = \sum_{j=1}^{n} d_{ij} = \sum_{j=1}^{n} t_j d_{ij}$$

(4)

Which means, in this model, the quantity of information output by health professional $i$, which is $d_i$, equals to the sum of information that all diabetes educators get from $i$.

The total information loss of health professionals is equal to the loss of information through transmission. To quantify the effect of the information, we need to define a variable $s$, which is the information concentration. The information concentration is the effect on $j$ placed per unit of information. $s$ is a nonnegative number. When $0 < s < 1$, dilution of information occurs, and people receiving this information may be less worried about their possibility of suffering from diabetes. In addition, when it is greater than 1, the information has a positive impetus on those who receive it, and they may be more careful after receiving the information.

The total effect maximization that the first tier can receive from the information output can be depicted as follows:

$$\text{Maximize } \sum_{j=1}^{n} s_{ij}^* d_{ij} - l_i(D^1) - \sum_{j=1}^{n} e_{ij}(d_{ij}) \quad d_{ij} \geq 0 \quad \text{for all } j \in Z^+$$

(5)

Figure 2. Network structure of health professional $i$. 
In the DITN, we assume that there is no information exchange between health professionals. They will not exchange their views or change their own idea because of others. In addition, one diabetes educator may gather information from different health professionals. Of course, their effect on the same diabetes educator is different. Equally, different diabetes educators have different thoughts regarding the same piece of information. So $s_{1ij}$ can be different with a different $i$ or $j$. The optimality conditions for all first-tier objects simultaneously can be expressed as the following variational inequality:

$$
\text{Determine } D^1 \in K_+^{mn} \text{satisfying }
\sum_{i=1}^m \sum_{j=1}^n \left[ \frac{\partial l_i(D^1)}{\partial d_{ij}} + \frac{\partial e_{ij}(d_{ij}^*)}{\partial d_{ij}} - s_{1ij}^* \right] \times [d_{ij} - d_{ij}^*] \geq 0 \quad \forall D^1 \in K_+^{mn}
$$

The Behavior of Diabetes Educators

Figure 3. shows the network structure of particular diabetes educator $j$. The diabetes educators, as the middle tier of the model, are involved in diagnostic information transmission both with the health professionals and the UD & PD. They receive the information from the health professionals. Then, via contact through their DITN, they educate those who may suffer from diabetes--UD & PD. UD & PD, who have not been to the hospital for a diagnosis to determine if they have diabetes, will decide whether they should go to the doctor to ensure that they are healthy. They may also change their lifestyle to avoid the possibility of getting sick. Diabetes educators are more like an information transfer station here, but they do not simply pass the information from the first tier to the third tier. During the transmission, the diabetes educators also add their own understanding and considerations to the information presented to the patients. So, for diabetes educator $j$, the amount of information they receive from the first tier is not equal to the amount of information that they pass down to the next tier. Therefore, the equation group (1) is necessary. $t_j$ is in some sense a conversion factor. It ensures the equilibrium state of the information flow.

$$
e_j = e_j(D^1) \quad \forall j \in Z^+ \quad (7)
$$

When receiving information, diabetes educator $j$ has their own reaction to the information. Therefore, the expected information concentration $s_j$ is different.
Therefore, the possible effect of one piece of information on diabetes educator \( j \) is denoted by \( s^*_{2j} \). There is also a best response that diabetes educator \( j \) wants to obtain. If we regard this as an optimization problem, it can be given by the following:

\[
\text{Maximize } \quad s^*_{2j} \sum_{k=1}^{p} d_{jk} - e_j(D^1) - \sum_{i=1}^{m} s^*_{1ij} d_{ij}
\]  

subject to

\[
\sum_{k=1}^{p} d_{jk} \leq \sum_{i=1}^{m} d_{ij}
\]

and the non-negativity constraints: \( d_{ij} \geq 0 \) and \( d_{jk} \geq 0 \) for all \( i \) and \( k \). Objective function (8) expresses that the difference between the effect of diabetes educators minus the error and loss of information transmission and the expected effect of health professionals should be maximized. We subtract the expected effect of the health professionals because both tiers cannot be simply added. Constraint (9) expresses that after the communication of \( t_j \), the amount of information taken in by \( j \) cannot be less than the \( j \) output.

We now consider the optimality conditions of the diabetes educators, assuming that each diabetes educator is faced with the optimization problem (8) subject to (9), and the non-negativity assumption on the variables. Here, we also assume that the diabetes educators will not change their information or ideas because of others. Here, we not only assume the optimal amounts of the outlet on the second tier but also the amount of information they receive from the health professionals. At equilibrium, the transmission of information between the tiers of network agents must be consistent.

Therefore, the optimality conditions for all the diabetes educators coincide with the solution of the variational inequality as follows:

Determine \( (D^1, D^2, \lambda^*) \in R_{+}^{mn+np+n} \) satisfying

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} \left[ \frac{\partial e_{ij}(D^1)}{d_{ij}} \right] s_{1ij}^* - \lambda_j^* \times \left[ d_{ij} - d_{ij}^* \right] + \sum_{j=1}^{n} \sum_{k=1}^{m} \left[ -s_{2j}^* + \lambda_j^* \right] 
\]

\[
\times \left[ d_{jk} - d_{jk}^* \right] + \sum_{j=1}^{n} \sum_{i=1}^{m} \left[ d_{ij}^* - \sum_{k=1}^{p} d_{jk}^* \right] \times \left[ \lambda_j - \lambda_j^* \right] \geq 0
\]

\[
\forall (D^1, D^2, \lambda) \in R_{+}^{mn+np+n}
\]

where \( \lambda_j \) is the Lagrange multiplier associated with constraint (9) for diabetes educator \( j \), \( \lambda \) denotes the \( n \)-dimensional column vector of all the multipliers, and \( D^2 \) denotes the \( n \times p \)-dimensional vector of information flows between the diabetes educators and UD & PD. In this derivation, as in the derivation of inequality (6), the information concentration is equal to the invariables in the equilibrium model, which are endogenous variables.

From the second term in inequality (10), we can see that, if UD & PD \( k \) receive information from diabetes educator \( j \), i.e., if \( d_{jk}^* \) is positive, then the expected information concentration of the information \( j \) output is precisely equal to \( \lambda_j^* \).
The Behavior of UD & PD

According to the information UD & PD receive from the diabetes educators, they will evaluate their potential for developing diabetes. Then, they shall decide whether they should attempt to improve their health to avoid developing diabetes or go to a doctor for an accurate diagnosis. Hence, the network structure of material $k$’s transactions is as depicted in Figure 4.

When passing the information on the disease to the lower tier, the diabetes educators intend to have an effect on the information receivers. Sometimes, the intention is to encourage the UD & PD to pay more attention to the disease and their health. The intended effect on the UD & PD $k$ is denoted by $s^*_3k$, which will also be endogenously determined in the model.

![Figure 4. Network structure of UD & PD $k$.](image)

A UD & PD $k$ is faced with what we call error and loss during information transmission, which may include, for example, an information transmission interruption and errors in understanding the information. We denote this information error and loss as $e_{jk}$, and we assume that it is continuous, positive, and of the general form as follows:

$$e_{jk} = e_{jk}(D^2) \quad \forall j \in Z^+, k \in Z^+$$

where $D^2$ is the $n \times p$-dimensional column vector of information flows between the diabetes educators and the UD & PD.

We denote the information demand of the third tier by $u_k$, assuming the function is as follows:

$$u_k = u_k(s_3) \quad \forall k \in Z^+$$

where $s_3$ is the $p$-dimensional column vector of the expected information concentration of the third tier.

The equilibrium conditions for $k$ at the third tier, hence, take the form: For all diabetes educators $j$, $j = 1, ..., n$,

$$s^*_2j + e_{jk}(D^2) \begin{cases} = s^*_2k & \text{if } d^*_jk > 0 \\ \geq s^*_3k & \text{if } d^*_jk = 0 \end{cases} \quad (s^*_3k) \begin{cases} = \sum_{j=1}^n d^*_jk & \text{if } s^*_3k > 0 \\ \leq \sum_{j=1}^n d^*_jk & \text{if } s^*_3k = 0 \end{cases}$$

In the equilibrium, conditions (13) will have to hold for all UD & PD $k$, and these, in turn, can also be expressed as a variational inequality problem, from (5) and (9), and given by:
Determine \((D^2*, s^*_3) \in R^{np+n}_+\),
\[
\sum_{j=1}^{n} \sum_{k=1}^{p} [s^*_{2j} + e_{jk}(D^2*) - s^*_{3k}] \times [d_{jk} - d^*_j] + \sum_{k=1}^{p} \left[ \sum_{j=1}^{n} d^*_j - u_k(s^*_3) \right] \\
\times [s^*_{3k} - s^*_{3k}] \geq 0 \quad \forall (D^2*, s^*_3) \in R^{np+n}_+
\]

(14)

In the context of the decisions of the third tier, we utilized expect functions rather than utility functions, as was the case for the health professionals and the diabetes educators, who were assumed to be faced with best reaction functions, which correspond to utility functions. Since we expect the number in the third tier to be much greater than that of the health professionals and the diabetes educators, we believe that the above formulation is the more natural and tractable one.

The Behavior of the DITN as a Whole

In equilibrium, the transmission of the information that the health professionals send to the diabetes educators must be equal to the transmission that the diabetes educators output. In addition, the amount of the information received by the UD & PD at the third tier must be equal to the output by diabetes educators. Furthermore, the equilibrium transmission and information concentration in the information transmission process must satisfy the sum of inequalities (6), (10), and (14), to formalize the agreements between the tiers. This is stated explicitly in the following definition:

Definition 2 (Information transmission network equilibrium). The equilibrium state of the information network occurs when the information flows between the three tiers of the decision-makers coincide and the information flows and information concentration (effect of information) satisfy the sum of the optimality conditions (6) and (10) and condition (14).

Then, we can have the following:

Theorem 3 (Variational inequality formulation). The equilibrium conditions of the DITN model with the constraint we set before, are equivalent to the solution of the variational inequality problem given by: Determine \((D^1*, D^2*, \gamma^*, s^*_3) \in \mathcal{K}\) satisfying:

\[
\text{(where } \mathcal{K} = \{(D^1, D^2, \lambda, s_3) / (D^1, D^2, \lambda, s_3) \in K^{mn+np+n}_+\}) \text{: Proof. See Appendix C)}
\]
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} \left[ \frac{\partial l_i(D^1*)}{\partial d_{ij}} + \frac{\partial e_{ij}(d^*_i)}{\partial d_{ij}} + \frac{\partial e_{ij}(D^1*)}{\partial d_{ij}} - \lambda^*_j \right] \times [d_{ij} - d^*_j] \\
+ \sum_{j=1}^{n} \sum_{k=1}^{p} [e_{jk}(D^2*) + \lambda^*_j - s^*_3k] \times [d_{jk} - d^*_jk] \\
+ \sum_{j=1}^{n} \sum_{i=1}^{m} d^*_ij - \sum_{k=1}^{p} d^*_jk \right] \times [\lambda_j^* - \lambda^*_j] \\
+ \sum_{k=1}^{p} \left[ \sum_{j=1}^{n} d^*_jk - u_k(s^*_3) \right] \times [s^*_{3k} - s^*_3k] \geq 0 \quad \forall (D^1, D^2, \lambda, s_3) \in \mathcal{K}
\]

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DISCUSSION AND CONCLUSION

Discussion

Numerical example see Appendix D.

Through the results of numerical examples, we can see that the increased number of diabetes educators in the second tier resulted in a reduction in the amount of information sent from health professionals to diabetes educators. In addition, the information output from diabetes educators to UD & PD is also reduced. The total amount of medical information transmission in the entire network is almost unchanged. From the unit information concentration, the increased diabetes educators causes each tier to have a corresponding increase in the information concentration. Therefore, when the input and output diagnostic information of a single object is reduced, whether to a diabetes educator or UD & PD, the reference value of a certain piece of information will rise.

On the other hand, the increase in the number of health professionals resulted in a reduction in the information output from a health professional to a single diabetes educator, and a reduction in the total amount of information output across the network. As a result of the authority and professionalism of health professionals, the increase in the number of health professional caused a significant increase in the information concentration. The information concentrations $s_{31}$, $s_{32}$ and $s_{33}$ of the UD & PD are greater than $s_{31}$, $s_{32}$. We believe that the reduction in the amount of information transmission from health professionals leads to UD & PD to make decisions with more consideration of information from the second tier.

The quantity relations and trends between three examples above see Appendix E.

Conclusion

Determining the amount of information and the concentration given to each unit of information is the goal of this model. The equilibrium point consists of the information content and information concentration and other variables which constitute the transmission of medical information that achieves the optimal effect of the whole network. This allows the UD & PD to give adequate attention to diabetes.

Diabetes is a chronic disease with a long period of progression. Information transmission by diabetes educators indifferent circumstances will vary. A one-time medical assessment is not sufficient for the UD & PD. Previous experience will have an impact on subsequent decisions. In other words, the transmission of diabetes information and the corresponding decision will be affected by previous medical experience. The relationship between these factors and their relationship with time requires follow up research.

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