Interval-based Possibilistic Description Logic Programs for the Semantic Web

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Abstract. Integration ontologies and rules has become a central topic in the Semantic Web. In order to deal with uncertainty and inconsistent information, possibilistic description logic program has been investigated in recent years. However, possibilistic description logic program also cannot well model a great deal of real-world problems, because the accurate degrees associated with axioms and atoms are usually difficult to provide for experts. To address this problem, we further extend possibilistic description logic programs so that they can deal with inaccurate degrees associated with axioms and atoms. Therefore, we propose tightly coupled interval-based possibilistic description logic programs under possibilistic answer set semantics, which are a tight integration of disjunctive logic programs, interval-based possibilistic logics and possibilistic description logics. First of all, we define the syntax and semantics. Then, we show some semantic properties. Furthermore, we present three reasoning problems, and present some algorithms to solve these reasoning problems.

Introduction

As an extension of the current World Wide Web, the Semantic Web aims to help computers to understand and process web information automatically [1]. Now, the integration ontologies and rules has become a central topic in the Semantic Web. In fact, standard ontology language is based on Description Logic[2], and the existing proposals for a rule language for use in the Semantic Web originate from Logic Programming[3]. Eiter et al proposed description logic programs under answer set semantics[4]. Subsequently, Lukasiewicz presented a new method to description logic programs under the answer set semantics [5]. Yidong Shen proposed well-justified FLP answer set for description logic programs [6].

Unfortunately, a great deal of real-world problems cannot be modelled by description logic programs, because crisp description logic programs cannot model uncertain and inconsistent information. Nevertheless possibilistic logic can represent and reason on uncertain, incomplete and inconsistent knowledge [7]. Dubois et al. defined the syntax and semantic of possibilistic logic, and also provided some reasoning problems[8]. Dellunde et al. present several fuzzy logics trying to capture different notions of necessity for Gödel logic formulas[9]. Benferhat et al. present an interval-based possibilistic logic[10]. Subsequently, Dubois propose a generalized possibilistic logic[11]. In fact, many researchers have focused their study on possibilistic description logics in recent years [12]. Subsequently, Dubois proposed a new possibilistic description logic [13]. Moreover, Guilin proposed a new possibilistic description logics [14]. Furthermore, he provided a possibilistic description logic reasoner PossDL[15]. Moreover, Lesot et al. improved tableau algorithm and proposed a new algorithm to compute the inconsistency degree of a possibilistic DL knowledge base[16]. We propose an interval-based possibilistic description logic IP-SHOIN(D)[17].

Therefore, this paper aims to further extend possibilistic description logic programs such that they can represent uncertain information with intervals. In this paper, we propose tightly coupled interval-based possibilistic description logic program (or simply interval-based possibilistic dl-program) under interval-based possibilistic answer set semantics, which combines possibilistic description logic program with an interval-based possibilistic logic. Firstly, we define the syntax and semantics
of interval-based possibilistic dl-program. Then we show some semantic properties. Finally, we define three reasoning problems, and present some algorithms to implement these reasoning problems.

**Interval-based Possibilistic Description Logic Programs**

**Syntax**

Let $\Phi$ be a function-free first-order vocabulary with nonempty finite sets of constant symbols $\Phi_{c}$ and predicate symbols $\Phi_{p}$. Suppose $\Phi_{c} \subseteq I_{c} \cup I_{p}$, thus every ground atom made from $C_{\Lambda}$, $R_{\Lambda}$, $R_{D}$, and $\Phi_{c}$ can be interpreted in the description logic component. Let $X$ be a set of variables. A term is either a variable from $X$ or a constant symbol from $\Phi_{c}$. An atom is an expression of the form $h(t_{1},...,t_{n})$, where $h$ is a predicate symbol of arity $n \geq 0$ from $\Phi_{p}$, and $t_{1},...,t_{n}$ are terms. We use $M$ to denote a set of atoms. A literal is an atom or a negated atom not $a$, where $a \in M$. Moreover, we use $L=\alpha, \beta \in [0,1]$ to denote real number based interval and restrict that $\alpha > 0$, and use $L$ to denote the set of intervals. In the following, we define some operations on intervals.

**Definition 2.1.** The operations on intervals are defined as follows:

1. **Max operation** $\oplus$: $\oplus \{L_{1}, L_{2},..., L_{n}\}=\{\max\{\alpha_{1}, \alpha_{2},..., \alpha_{n}\}, \max\{\beta_{1}, \beta_{2},..., \beta_{n}\}\}$, where $L_{1}=\alpha_{1}, \beta_{1}$, $L_{2}=\alpha_{2}, \beta_{2}$, ..., and $L_{n}=\alpha_{n}, \beta_{n}$ are intervals.

2. **Min operation** $\ominus$: $\ominus \{L_{1}, L_{2},..., L_{n}\}=\{\min\{\alpha_{1}, \alpha_{2},..., \alpha_{n}\}, \min\{\beta_{1}, \beta_{2},..., \beta_{n}\}\}$, where $L_{1}=\alpha_{1}, \beta_{1}$, $L_{2}=\alpha_{2}, \beta_{2}$, ..., and $L_{n}=\alpha_{n}, \beta_{n}$ are intervals.

3. **Compare operation** $>_{L}$: $L_{1}>L_{2}$ if and only if $\alpha_{1} > \beta_{2}$, where $L_{1}=\alpha_{1}, \beta_{1}$ and $L_{2}=\alpha_{2}, \beta_{2}$ are intervals.

4. **Subsume operation** $\ll$: $L_{1}\ll L_{2}$ if and only if $\alpha_{1} \geq \alpha_{2}$ and $\beta_{1} \leq \beta_{2}$, where $L_{1}=\alpha_{1}, \beta_{1}$ and $L_{2}=\alpha_{2}, \beta_{2}$ are intervals.

5. **Reverse operation** $\sim$: $1-\sim L=[1-\beta, 1-\alpha]$, where $L=\alpha, \beta$ is an interval.

**Definition 2.2.** An interval-based possibilistic atom is a pair $aL_{PM}$, which denotes that the certainty degree of the atom $a$ is one of the elements of the interval $L$. Moreover, we define the classical projection $\ast$ and necessity degree projection $\cdot$ as follows: $p^{\ast}=a$, $d(p)=L$.

In this paper, we use $PM$ to denote a set of possibilistic atoms in which every atom $a$ occurs at most one time in $PM$ and always with a strictly positive certainty degree, that is to say, $\forall a \in M, |<a, L> \in PM| \leq 1$.

**Definition 2.3.** An interval-based possibilistic disjunctive rule (or simply interval-based possibilistic rule) $r$ is of the form $r=\langle a_{1}, b_{1}, \ldots, b_{n} \rangle \leftarrow b_{1} \land \ldots \land b_{n} \land \neg b_{1_{n}} \land \ldots \land \neg b_{n_{n}}, L_{>}$, where $k \geq 0$, $l \geq 0$, $k+l > 0$, $\{a_{1}, \ldots, a_{l}, b_{1}, \ldots, b_{n}\} \subseteq M$, and $L \in L$. Moreover, we define the classical projection $\ast$ and the necessity degree projection $\cdot$ of the possibilistic rule $r$ as follows: $d(r)=L$

**Definition 2.4.** An interval-based possibilistic disjunctive program (or simply interval-based possibilistic program) $IP$ is a finite set of interval-based possibilistic rules. Let $IP^{\ast}=\{r^{\ast} | r \in IP\}$, then, $IP$ is normal interval-based possibilistic program iff $k=1$ for all rules in $IP^{\ast}$; $IP$ is a positive interval-based possibilistic program iff $n=1$ for all rules in $IP^{\ast}$.

**Definition 2.5.** An interval-based possibilistic description logic program (for short, interval-based possibilistic dl-program) $IKB=(IL, IP)$ includes an interval-based possibilistic description logic knowledge base $IL$ and an interval-based possibilistic program $IP$.
Semantics

A term is ground iff it includes only constant symbols from $\Phi_c$. An atom $h(t_1, \ldots, t_n)$ is ground iff all terms $t_1, \ldots, t_n$ are ground.

**Definition 2.6.** An interval-based possibilistic atom $p = \langle a, L \rangle$ is a ground atom iff the atom $a$ is ground. An interval-based possibilistic rule $r$ is ground rule iff all atoms $a_i, \ldots, b_i, \ldots, b_n$ are ground atoms.

**Definition 2.7.** A ground instance of an interval-based possibilistic rule $r$ is an interval-based possibilistic ground rule $r' = \langle a'_i \vee \cdots \vee a'_i \leftarrow b'_i \wedge \cdots \wedge \text{not } b'_i \wedge \cdots \wedge \text{not } b'_i, L \rangle$, where $a'_i, \ldots, a'_i, b'_i, \ldots, b'_i, \ldots, b'_i$ are obtained by substituting constant symbols from $\Phi_c$ for every variable appearing in $a_i, \ldots, a_i, b_i, \ldots, b_i, \ldots, b_i$ respectively. A ground program of an interval-based possibilistic program IP is a set of all ground instances of interval-based possibilistic rules in IP.

In this paper, we use $\text{PossG}(P)$ to denote all ground programs of an interval-based possibilistic program IP.

**Definition 2.8.** The possibilistic Herbrand base relative to $\Phi$, written as $\text{PHB}_\Phi$, is a set of all interval-based possibilistic ground atom $\{p_1, \ldots, p_n\}$, for every interval-based possibilistic ground atom $p_i = \langle h_i(t_{i1}, \ldots, t_{in}), L_i \rangle$, $i = 1, 2, \ldots, n$, $h_i$ is a predicate symbol of arity $n \geq 0$ from $\Phi_p$, and $t_{i1}, \ldots, t_{in}$ are constant symbols from $\Phi_c$, and $L_i \in \mathcal{L}$.

**Definition 2.9.** An interval-based possibilistic interpretation $I = \{p'_1, \ldots, p'_n\}$ relative to IKB is a subset of $\text{PHB}_\Phi$.

**Definition 2.10.** An interval-based possibilistic interpretation $I$ is an interval-based possibilistic model of $p = \langle a, L \rangle$, denoted $I \models \langle a, L \rangle$, if and only if $\langle a, L \rangle \in I$ or there exists an interval-based possibilistic ground atom $\langle a, L' \rangle \in I$ such that $L' \ll L$. An interval-based possibilistic interpretation $I$ is an interval-based possibilistic model of an interval-based possibilistic ground rule $r$, denoted by $I \models r$, if and only if, $I \models \langle a, \forall \{L_i, L_{i1}, \ldots, L_{in}\} \rangle$ for some $a \in H(r')$, if $I \models \langle h_i(L_i), \ b_i \in B'(r') \rangle, L_i \in \mathcal{L}$, $i = 1, 2, \ldots, l$, and $I \models \langle b_j(L_j) \rangle, b_j \in B'(r') \rangle, L_j \in \mathcal{L}$, $j = l + 1, l + 2, \ldots, n$. An interval-based possibilistic interpretation $I$ is an interval-based possibilistic model of IP, denoted by $I \models IP$, if and only if $I \models r$ for all $r \in \text{PossG}(IP)$.

**Definition 2.11.** An interval-based possibilistic interpretation $I$ is an interval-based possibilistic model of $IL$, denoted $I \models IL$, if and only if there exists an interval-based possibilistic distribution $\pi$ for knowledge base $IL \cup I$ such that $\pi \models IL \cup I$. $I$ is an interval-based possibilistic model of IKB, denoted $I \models IKB$, if and only if $I \models IL$ and $I \models IP$.

There may be many interval-based possibilistic models for an interval-based possibilistic d-program $IKB = (IL, IP)$. Let $I_1$, $I_2$ be two interval-based possibilistic models of IKB, then $I_1 \cap I_2 = \{a, [\min\{\alpha_i, \alpha_i\}, \max\{\beta_i, \beta_i\}] > \alpha_i, [\alpha_i, \beta_i] \in I_1, [\alpha_i, \beta_i] \in I_2\} \iff I_1 \subseteq I_2$ and only if $I_1 \subseteq I_2$, or $I_2 \subseteq I_1$ and for any $a \in I_1$, if $a \not\in I_2$, $a \not\in I_2$, and $a \not\in I_2$, then $I_2 \subseteq I_1$.

**Definition 2.12.** An interval-based possibilistic interpretation $I$ is a least interval-based possibilistic model of IKB, if and only if there does not exist an interval-based possibilistic model $I'$ of IKB, such that $I' \ll I$.

**Definition 2.13.** An interval-based possibilistic reduction for IP is defined as follows: $IP_{IP'} = \{\exists \mathcal{A} \cap PM' \leftarrow B', L > r \leftarrow A \leftarrow B' \wedge \text{not } B', L \in P, A \cap PM' \neq \emptyset, B' \cap PM' = \emptyset, B' \subseteq PM'\}$. Moreover, an interval-based possibilistic reduction for IKB is $IKB_{IP'} = (IL, IP_{IP'})$.

**Definition 2.14.** Let $I$ be an interval-based possibilistic interpretation relative to IKB such that $I'$ is an answer set of $IKB'$. Then $I$ is an interval-based possibilistic answer set of IKB if and only if $I$ is a least interval-based possibilistic model of IKB such that $IKB' \models I$.
Semantic Properties

**Theorem 3.1.** Let \( \Phi \) be an interval-based possibilistic answer set of \( KB=(IL, IP) \) if and only if \( I \) is an interval-based possibilistic model of \( IP \), where \( IL=\emptyset \).

Proof. It is known that \( I \) is an interval-based possibilistic answer set of \( IKB \) if and only if \( I \) is a least interval-based possibilistic model of \( IKB \), iff \( I \models IL \) and \( I \models \Phi \) for all \( r \in PossG(IP) \). Because \( IL=\emptyset \), then \( I \) is an interval-based possibilistic model of \( IKB \), iff \( I \models \Phi \) for all \( r \in PossG(IP) \) iff \( I \) is an interval-based possibilistic model of \( IP \). Thus, \( I \) is a least interval-based possibilistic model of \( IKB \), iff \( I \) is a least interval-based possibilistic model of \( IP \). Moreover, \( IKB \models I \) iff \( IP \models I \).

**Theorem 3.2.** Let \( \langle \varphi, L \rangle \) be an interval-based possibilistic ground atom of \( PHB_\alpha \). Then for all interval-based possibilistic answer set \( I \) of \( IKB \), \( I \models \langle \varphi, L \rangle \) if and only if for all interval-based possibilistic distribution \( \pi \) such that \( \pi \models IL \cup PossG(IP) \), \( \pi \models \langle \varphi, L \rangle \).

Proof. Because \( IKB \) is a positive interval-based possibilistic dl-program, then the set of all possibilistic answer set of \( IKB \) is equivalent to the set of all interval-based possibilistic model of \( KB \). Moreover, for \( \langle \varphi, L \rangle \in PHB_\alpha \), for all least interval-based possibilistic model of \( IKB \), \( I \models \langle \varphi, L \rangle \) iff for all interval-based possibilistic model of \( IKB \), \( J \models \langle \varphi, L \rangle \). So, for all interval-based possibilistic answer set \( I \models \langle \varphi, L \rangle \) iff for all interval-based possibilistic model of \( KB \), \( \models \langle \varphi, L \rangle \). Now, we need to prove that for all interval-based possibilistic model \( J \) of \( IKB \) \( J \models \langle \varphi, L \rangle \) iff for all interval-based possibilistic distribution \( \pi \) such that \( \pi \models IL \cup PossG(IP) \), \( \pi \models \langle \varphi, L \rangle \).

\((\Rightarrow)\) Suppose that for all interval-based possibilistic model \( J \) of \( IKB \), \( J \models \langle \varphi, L \rangle \). Let \( \pi \) be any interval-based possibilistic distribution such that \( \pi \models IL \cup PossG(IP) \). Now, we define an interval-based possibilistic interpretation \( I' \subseteq PHB_\alpha \) such that \( I' \models \langle \varphi, L' \rangle \) iff \( \pi \models \langle \varphi, L' \rangle \). Let \( I' = IL \cup I' \), then \( I' \) is an interval-based possibilistic model of \( IL \). Because \( \pi \models PossG(IP) \), then \( \pi \models r \) for all \( r \in PossG(IP) \). Thus, \( I' \models r \) for all \( r \in PossG(IP) \). So, \( I' \) is also an interval-based possibilistic model of \( IP \). Therefore, \( I' \) is an interval-based possibilistic model of \( IKB \). According to \( I' \models \langle \varphi, L' \rangle \). So, \( \pi \models \langle \varphi, L \rangle \). Therefore, for all interval-based possibilistic distribution \( \pi \) such that \( \pi \models IL \cup PossG(IP) \), \( \pi \models \langle \varphi, L \rangle \).

\((\Leftarrow)\) Suppose that for all interval-based possibilistic distribution \( \pi \) such that \( \pi \models IL \cup PossG(IP) \), \( \pi \models \langle \varphi, L \rangle \). Let \( I \subseteq PHB_\alpha \) be any interval-based possibilistic model of \( IKB \). So, \( I \models IL \) and \( I \models r \) for all \( r \in PossG(IP) \). Thus, there exists an interval-based possibilistic \( \pi' \) such that \( \pi' \models IL \cup I \). So, \( \pi' \models IL \), \( \pi' \models I \). Moreover, \( \pi' \models r \) for all \( r \in PossG(IP) \), and thus \( r \in PossG(IP) \). So, \( \pi' \models IL \cup PossG(IP) \). According to known condition, \( \pi' \models \varphi, L \). Thus, \( \varphi, L \models I \), or there exists an interval-based possibilistic ground atom \( \varphi, L' \models I \), such that \( L' \models L \). So, \( I \models \langle \varphi, L \rangle \). Therefore, for all interval-based possibilistic answer set \( I \models \langle \varphi, L \rangle \).

**Theorem 3.3.** Let \( \langle \varphi, L \rangle \) be an interval-based possibilistic ground atom of \( PHB_\alpha \), and \( IP=\emptyset \). Then for all interval-based possibilistic answer set \( I \) of \( IKB \), \( I \models \langle \varphi, L \rangle \) iff for all interval-based possibilistic distribution \( \pi \) such that \( \pi \models IL \), \( \pi \models \langle \varphi, L \rangle \).

Proof. It is obvious.

**Reasoning Problems for Interval-Based Possibilistic Description Logic Programs**

Let \( IKB=(IL, IP) \) be an interval-based possibilistic dl-program and \( L \) be an interval. Then the classical DL knowledge base associated with \( IL \) is \( IL' = \{ \varphi \models \langle \varphi, L \rangle \models IL \} \), and the \( L \)-cut set of \( IL \) is \( IL_{\leq L} = \{ \varphi \models \langle \varphi, L \rangle \models IL, and L > L \} \). The classical logic program associated with \( IP \) is \( IP' = \{ \langle r \rangle \models r \models IP \} \), and the \( L \)-cut set of \( IP \) is \( IP_{\geq L} = \{ r \models d(r) \models L \} \). The classical dl-program associated with \( IKB \) is \( IKB'=(IL', IP') \), and the \( L \)-cut set of \( IKB \) is \( IKB_{\leq L}=(IL_{\leq L}, IP_{\leq L}) \). Moreover, \( IKB \) is consistent if and only if \( IKB' \) is consistent.
Definition 4.1. Let \( IKB=(IL, IP) \) be an interval-based possibilistic dl-program. Then a possibilistic description logic program KB is compatible with \( I \) if and only if there exists a bijection function \( f : IKB \leftrightarrow KB \), such that for any \( \langle \phi, L \rangle \in IKB \), \( f(\langle \phi, L \rangle) = \langle \phi, \delta \rangle \in KB \), \( \delta \in L \), and for any \( \langle \phi, \alpha \rangle \in KB \), \( f(\langle \phi, \alpha \rangle) = \langle \phi, L \rangle \in IKB \), \( \alpha \in L \).

For any interval-based possibilistic dl-program \( IKB \), we can obtain a compatible possibilistic description logic program \( KB = \langle \phi, \delta \rangle | \langle \phi, L \rangle \in IKB \) and \( \delta \in L \). Thus, we use \( CPB(IKB) \) to denote the infinite set of all possibility description logic program knowledge bases that are compatible with \( IKB \).

Definition 4.2. Let \( IKB=(IL, IP) \) be an interval-based possibilistic dl-program. Then a lower bound compatible possibilistic description logic program knowledge base \( KB_{lb} \) and an upper bound compatible possibilistic description logic program knowledge base \( KB_{ub} \) are defined as follows:

\[
KB_{lb} = \{ \langle \phi, \alpha \rangle | \phi, [\alpha, \beta] \in IKB \}, \quad KB_{ub} = \{ \langle \phi, \beta \rangle | \phi, [\alpha, \beta] \in IKB \}.
\]

Definition 4.3. Let \( IKB=(IL, IP) \) be an interval-based possibilistic dl-program. Then the interval-based consistency degree of \( IKB \), denoted by \( Icon(IKB) \), is defined as follows:

\[ Icon(IKB) = \{ Icon(KB) | KB \in CPB(IKB) \}. \]

Theorem 4.4. Let \( IKB=(IL, IP) \) be an interval-based possibilistic dl-program. Then

\[
Icon(IKB) = \left[ \min_{KB \in CPB(IKB)} Icon(KB), \max_{KB \in CPB(IKB)} Icon(KB) \right].
\]

Proof. According to Definition 4.1, for any \( KB \in CPB(IKB) \), \( Icon(KB) \in Icon(IKB) \). Thus,

\[
\forall KB' \in CPB(IKB), \quad \min_{KB \in CPB(IKB)} Icon(KB) \leq Icon(KB') \leq \max_{KB \in CPB(IKB)} Icon(KB).
\]

Therefore, \( Icon(IKB) = \left[ \min_{KB \in CPB(IKB)} Icon(KB), \max_{KB \in CPB(IKB)} Icon(KB) \right] \).

Theorem 4.5. Let \( IKB=(IL, IP) \) be an interval-based possibilistic dl-program, and \( KB_{lb} \) be a lower bound compatible possibilistic description logic program knowledge base of \( IKB \), and \( KB_{ub} \) be an upper bound compatible possibilistic description logic program knowledge base of \( IKB \). Then \( Icon(KB) = \{ Icon(KB_{lb}), Icon(KB_{ub}) \} \).

Proof. \( \forall KB \in CPB(IKB), KB = \langle \phi, \delta \rangle | \phi, [\alpha, \beta] \in IKB \) and \( \delta \in [\alpha, \beta] \). So, the consistency degree of \( KB \) is

\[ Icon(KB) = \min([\delta | KB_{lb} is consistent]) \].

Moreover, \( KB_{lb} = \{ \langle \phi, \alpha \rangle | \phi, [\alpha, \beta] \in IKB \} \), \( KB_{ub} = \{ \langle \phi, \beta \rangle | \phi, [\alpha, \beta] \in IKB \} \). Thus, \( Icon(KB_{lb}) = \min([\alpha | KB_{lb} is consistent]) \), \( Icon(KB_{ub}) = \min([\beta | KB_{ub} is consistent]) \). Because \( \alpha \leq \delta \leq \beta \), \( \forall KB \in CPB(IKB), Icon(KB_{lb}) \leq Icon(KB) \leq Icon(KB_{ub}) \). Therefore, \( \max_{KB \in CPB(IKB)} Icon(KB) = Icon(KB_{ub}) \).

Theorem 4.6. Let \( IKB=(IL, IP) \) be an interval-based possibilistic dl-program, \( Icon(IKB) \) be an interval-based consistency degree of \( IKB \). Then,

1) \( \forall KB \in CPB(IKB), KB_{<Icon(KB)} \subseteq KB_{>Icon(KB)}. \)

2) \( KB_{Icon(KB)} \) is consistent.

Proof. 1) \( KB_{<Icon(KB)} = \{ \phi | \phi, L \in IKB \) and \( L > Icon(KB) \}. \) For any \( KB \in CPB(IKB) \), we have \( KB = \{ \langle \phi, \delta \rangle | \phi, L \in IKB \) and \( \delta \in L \} \). So, \( Icon(KB) = \min([\delta | KB_{<Icon(KB)} is consistent]) \), \( KB_{<Icon(KB)} = \{ \langle \phi, \delta \rangle | \phi, \delta \in IKB \) and \( \delta > Icon(KB) \}. \) According to Definition 4.3, for any \( KB \in CPB(IKB) \), \( Icon(KB) \in Icon(IKB) \). Thus, \( \forall \phi, L \in IKB \) and \( L > Icon(KB) \), there exists a possibilistic atom \( \langle \phi, \delta \rangle \in KB \) such that \( \delta \in L \) and \( \delta > Icon(KB). \) So, \( \forall \phi \in KB_{>Icon(KB)} \), we have \( \phi \in KB_{<Icon(KB)}. \) Therefore, \( \forall KB \in CPB(IKB), KB_{>Icon(KB)} \subseteq KB_{<Icon(KB)}. \)
2) According to the definition of consistency degree of interval-based possibilistic dl-program knowledge base, we can obtain, \( \forall KB \in \text{CPb}(IKB), K^c B_{\text{icon}(IKB)} \) is consistent. Therefore, \( IKB_{\text{icon}(IKB)} \) is consistent.

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<tr>
<th>Algorithm ICONIP</th>
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<tr>
<td><strong>Input:</strong> an interval-based possibilistic dl-program knowledge base IKB.</td>
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<tr>
<td><strong>Output:</strong> an interval-based consistency degree of IKB, i.e. ( \text{Icon}(IKB) ).</td>
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<tr>
<td>1. Let ( KB_{lb} = \langle \phi, \alpha \rangle &lt; \phi, [\alpha, \beta] &gt; \in IKB ).</td>
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<tr>
<td>2. Compute the consistency degree of ( KB_{lb} ), i.e. ( \text{Icon}(KB_{lb}) ).</td>
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<tr>
<td>3. Let ( KB_{ub} = \langle \phi, \beta \rangle &lt; \phi, [\alpha, \beta] &gt; \in IKB ).</td>
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<tr>
<td>4. Compute the consistency degree of ( KB_{ub} ), i.e. ( \text{Icon}(KB_{ub}) ).</td>
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<tr>
<td>5. Let ( \text{Icon}(IKB) = [\text{Icon}(KB_{lb}), \text{Icon}(KB_{ub})] ).</td>
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<tr>
<td>6. Return ( \text{Icon}(IKB) ).</td>
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Figure 1. Algorithm ICONIP.

According to the above theorems, we propose an algorithm ICONIP to compute an interval-based consistency degree of an interval-based possibilistic dl-program knowledge base IKB as shown in Figure 1. The main idea of this algorithm is to compute the consistency degree of two compatible possibilistic dl-program knowledge bases. Firstly, we obtain a lower bound compatible possibilistic dl-program knowledge base \( KB_{lb} \) of IKB, and an upper bound compatible possibilistic dl-program knowledge base \( KB_{ub} \) of IKB. Then we compute the consistency degree of \( KB_{lb} \), i.e. \( \text{Icon}(KB_{lb}) \), and the consistency degree of \( KB_{ub} \), i.e. \( \text{Icon}(KB_{ub}) \). Finally, we get an interval-based consistency degree of IKB, \( \text{Icon}(IKB) = [\text{Icon}(KB_{lb}), \text{Icon}(KB_{ub})] \).

**Definition 4.7.** An interval-based possibilistic axiom or atom \( < \phi, L > \) is called credutions possibilistic consequence of IKB, denoted by \( IKB \models_{c} < \phi, L > \), if and only if, there exists an interval-based possibilistic answer set \( I \) of IKB such that \( I \models < \phi, L > \).

**Definition 4.8.** An interval-based possibilistic axiom or atom \( < \phi, L > \) is called skeptical possibilistic consequence of IKB, denoted by \( IKB \models_{s} < \phi, L > \), if and only if, for all interval-based possibilistic answer set \( I_1, I_2, ..., I_m \) of IKB such that \( I_1 \supseteq I_2 \supseteq \ldots \supseteq I_m \models < \phi, L > \).

**Conclusion**

In this paper, we combine description logic program with interval-based possibilistic logic, and propose an interval-based possibilistic description logic program. Firstly, we provide the syntax and semantics of interval-based possibilistic dl-program, the concepts of interval-based description logics are used as unary predicates of possibilistic disjunctive programs and the roles of interval-based description logics are used as binary predicates possibilistic disjunctive programs. Secondly, we present some semantic properties of interval-based possibilistic dl-program under interval-based
possibilistic answer set semantics. We prove that the interval-based possibilistic answer set semantics of an interval-based possibilistic dl-program faithfully extends the possibilistic semantics of an interval-based possibilistic program and interval-based possibilistic description logic. Finally, we present three reasoning problems: computing consistency degree, credutions possibilistic consequence and skeptical possibilistic consequence, and propose some algorithms to solve these reasoning problems. In a word, possibilistic dl-program can well represent and reason a great deal of real-word problems. An interesting topic of future research is to implement of the presented approach. Another interesting issue is to extend interval-based possibilistic dl-programs by a new semantics.

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