Numerical Method of Radiation Impedance Calculation in Enclosed Space

Rui TANG¹,²,³, Xin FENG¹,², Yi-ming ZHANG¹,²,³ and Qi Li¹,²,³

¹Acoustic Science and Technology Laboratory, Harbin Engineering University, Harbin 15001, China
²Key Laboratory of Marine Information Acquisition and Security (Harbin Engineering University), Ministry of Industry and Information Technology; Harbin 15001, China
³College of Underwater Acoustic Engineering, Harbin Engineering University, Harbin 15001, China

*Corresponding author

Keywords: Radiation impedance, Numerical calculation, Enclosed space.

Abstract. A numerical method for calculating the radiation impedance of sound sources in enclosed space is established in this paper. The correctness of the proposed method is verified by analytical results. Primarily, the radiation impedance of a pulsating sphere sound source in the free field and the enclosed space is analyzed using analytical calculation method. Furthermore, the radiation impedance of a pulsating ball sound source is calculated by acoustic finite element software Actran using the proposed numerical method. The research shows that the method of calculating the sound source radiation impedance by Actran is suitable for the analysis of the acoustic impedance characteristics of underwater sound sources. The numerical method established in this paper provides a useful numerical method to analyze the radiation impedance characteristics of underwater sound sources with complex geometric structure.

Introduction

Acoustic radiation impedance refers to the impedance produced by the vibration of sound sources, and it describes the reaction between the medium and the vibration surface. Knowing the radiation impedance can help understand the acoustic mechanism of sound source radiation, and provide theoretical basis for the measurement for the radiation sound power of the sound sources. Malhi¹ used the finite element software COMSOL Multiphysics to calculate the radiation impedance of a circular plate with cuts and mounted in an infinite baffle. J.L. Davy¹²³⁴ studied the analytical derivation of the sound source radiation impedance of a rectangular plate and extended the application scope of the equation⁵⁶. In the past studies, the analysis of the radiation impedance of piston source is common, but for other types of sound sources, the research of radiation impedance is relatively rare, because it is difficult to obtain the radiation impedance of the complex sources. A numerical method to calculate the radiation impedance of sound sources is proposed in this paper. The radiation impedance of a pulsating sphere sound source is calculated using the software Actran. Since the proposed numerical method is not subject to the shape and material of the sound sources and the sound field boundaries, the proposed method can be used to analyze the radiation impedance characteristics of underwater sound sources with complex geometric structure.

Analysis of Radiation Impedance of Pulsating Sphere Sound Source in the Free Field

Analytical Calculation of the Radiation Impedance of a Pulsating Sphere Sound Source in the Free Field

On the one hand, acoustic waves are emitted when the sound source vibrating in the medium, on the other hand, the sound source is also in the sound field of its own, which is bound to be influenced by the counterforce of the sound field. This counterforce is equivalent to a force impedance added to the original mechanical vibration system, that is called the radiation impedance. The analytical calculation formula of the radiation impedance in the free field can be written as
### Numerical Calculation of the Radiation Impedance of a Pulsating Sphere Sound Source in the Free Field

The flow chart of the numerical calculation of the radiation impedance is shown in Figure 1.

![Flow chart of numerical calculation](image)

The numerical calculation model of the radiation impedance of a pulsating sphere sound source is shown in Figure 2. The model is axisymmetric, so the modal is built in two-dimensional space. The sound source is located at the center of the model, with a radius of 0.1 m and the transmitting frequency is 100-12000 Hz. The radius of fluid domain is 1 m, and the acoustic infinite element simulation free field boundary is added to the surface of the fluid region.

![Image 2](image)

After the post-processing, the complex sound pressure at the measuring point can be obtained and the sound pressure of each point is approximated as the local sound pressure. Through the complex sound pressure, vibration velocity of the model surface and the definition of the radiation impedance, the calculation formula of radiation impedance is obtained as

$Z_r = R_r + jX_r = \rho_0 c_0 \left( \frac{k^2 r_0^2}{1 + k^2 r_0^2} \right) S_0 + j \rho_0 c_0 \left( \frac{kr_0}{1 + k^2 r_0^2} \right) S_0$  \hspace{1cm} (1)

Where $R_r$ is radiation resistance, $X_r$ is radiation reactance and $Z_r$ is radiation impedance.

According to formula (2), the numerical results can be obtained. The radiation impedance
calculated by numerical and analytical method is compared and the comparison results are shown in Figure 3.

![Figure 3](image)

Figure 3. The comparison of radiation impedance obtained by numerical calculation and analytical calculation.

It can be seen from Figure 3 that the comparison results are consistent, the radiation impedance obtained by numerical calculation and analytical calculation fit well when the frequency is below 3000Hz, and when the frequency is higher than 3000 Hz, there are some errors due to less measurement points and less grids. These errors can be improved by increasing the number of measuring points and grids.

Analysis of Radiation Impedance of Pulsating Sphere Sound Source in Enclosed Space

Analytical Calculation of the Radiation Impedance of a Pulsating Sphere Sound Source in Enclosed Space

The analytical calculation formula of the radiation impedance in closed space can be written as

\[
Z_r = R_r + jX_r = i4\pi a^2 \rho \omega \sum_{n=1}^{\infty} \frac{\phi_n^2(r_0)}{(k_n^2 - k_n^2)}V\Lambda_n
\]

(4)

Where \(k_n\) is wave number of \(n^{th}\) order normal mode, \(\phi_n(r)\) is velocity potential function, \(\Lambda_n = \frac{1}{8}\).

Numerical Calculation of the Radiation Impedance of a Pulsating Sphere Sound Source in Closed Space

The numerical calculation model of the radiation impedance of a pulsating sphere sound source is shown in Figure 4. The sound source is located at the center of the model, with a radius of 0.05 m, and the transmitting frequency is 100-2500 Hz. The size of fluid domain is 1.5×0.9×0.6 m³, and the absolute soft boundary is added to the surface of the fluid to simulate the enclosed space[7].

![Figure 4](image)
Through extracting the complex sound pressure and the complex velocity on the surface of the model, the calculation formula of radiation impedance is obtained as

$$Z_r = \sum_{j=1}^{n} \sum_{l=1}^{n} \int_{\theta_{j}}^{\theta_{j+1}} \int_{\phi_{l}}^{\phi_{l+1}} P_i \cdot r^2 \sin \phi d\phi d\theta$$

(5)

Where, $\theta_j = \left\{ \begin{array}{ll} \theta_j - 360 / m & j > 1 \\ 0 & j = 1 \end{array} \right.$

(6)

According to formula (5), the numerical results can be obtained. The radiation impedance value calculated by numerical and analytical method is compared, and the comparison results are shown in Figure 5.

![Normalization of radiation resistance](c)

![Normalization of radiation reactance](d)

Figure 5. The normalization comparison of radiation impedance.

It can be seen from Figure 5 that the comparison results are consistent. The correctness of the proposed method is verified.

**Conclusion**

The analytical radiation impedance expressions of pulsating sphere sound sources in the free field and closed space are derived in this paper. The radiation impedance of a pulsating sphere sound source is calculated using the proposed numerical method. The correctness is verified by the analytical procedure. The research shows that the numerical model of the radiant impedance established by Actran is suitable for the calculation and analysis of the acoustic impedance characteristics of underwater sound sources. The numerical method established in this paper provides a useful numerical method for the analysis the impedance characteristics of underwater sound sources with complex geometric structures.

**Acknowledgement**

This research was financially supported by the National Science Foundation of China under grant numbers 11504065.

**References**


sided specific radiation wave impedance of a finite rectangular panel, Inter-noise 2014, 43: 1-10.

of a finite rectangular panel excited by a plane sound wave, The 21st International Congress on


radiation impedance of a rectangular panel, 12 September 2015.

the sound field in a non-anechoic tank with elastic boundary, 5 August 2016.

[8] Tang Rui, Li Qi, The Investigation on Predicting and Measuring the Sound Radiation of
Underwater Shell Structure in Low Frequency, 24 April 2013.