Statistical Characteristics and Scintillation Level of Scattered Radio Waves in the Magnetized Plasma

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Abstract. Second order statistical moments of scattered radiation in the magnetized plasma is considered using modified smooth perturbation method taking into account both diffraction effects and polarization coefficients of both the ordinary and extraordinary waves. The broadening of the spatial spectrum and displacement of its maximum are obtained. Scintillation level of scattered waves is analyzed for different parameters characterizing anisotropic irregularities for the ionospheric F-region.

Introduction

At the present time peculiarities of electromagnetic (EM) wave propagation in randomly inhomogeneous media have been rather well studied [1]. The analysis of the statistical properties of small-amplitude electromagnetic waves that have passed through a turbulent plasma slab is very important in many practical applications associated with both natural and laboratory plasmas. Many excellent reviews and books of scintillation theory and observations in the ionosphere have been published [2,3] whereas statistical characteristics of scattered radiation in the turbulent magnetized plasma are less studied. The fluctuations in amplitude and phase (scintillation) of radio waves propagating through the ionosphere are caused by plasma irregularities in the electron density. Ionospheric scintillation models contain the worldwide climatology of the ionospheric plasma density irregularities that cause scintillation, coupled to a model for the effects of these irregularities on radio signals. A high priority given to the ionospheric scintillation study comes from its significant impact on satellite radio communications. Statistical characteristics and scintillation level of scattered ordinary and extraordinary waves in the collision magnetized plasma normal to the external magnetic field have been considered in [4-6] using modify smooth perturbation method.

In this paper, stochastic differential equation of the phase fluctuation has been obtained in the principle plane containing wave vector of an incident wave and the external magnetic field. Polarization coefficients and diffraction effects are taken into account. Numerical calculations are carried out for new spectral function combining anisotropic Gaussian and power-law spectral functions using experimental data applied to the F-region of ionosphere.

Formulation of the Problem

Let a plane EM wave with frequency $\omega$ be incident from vacuum on a semi-infinite slab of turbulent collision magnetized plasma. We choose a Cartesian coordinate system such that $XY$ plane is the vacuum-plasma boundary, $Z$ axis is directed in the plasma slab, $YZ$ plane is generated by the external magnetic field vector $B_0$ and the wave vector $k$ of the refracted wave. Electric field $E$ in the turbulent magnetized plasma satisfies the differential equation:
\[
\left( \frac{\partial^2}{\partial x_j \partial x_i} - \Delta \delta_{ij} - k_0^2 \epsilon_j (\mathbf{r}) \right) \mathbf{E}_j (\mathbf{r}) = 0
\]  

(1)

Components of the dielectric permittivity of the collisionless magnetized plasma are described by the second rank tensor [7], \( \Delta \) is the Laplacian, \( \delta_{ij} \) is the Kronecker symbol, \( \mathbf{v}(\mathbf{r}) = \omega^2_p(\mathbf{r}) / \omega^2 \) plasma frequency \( \omega^2_p(\mathbf{r}) = [4\pi N(\mathbf{r}) e^2 / m]^2 \) is a random function of the spatial coordinates, \( N(\mathbf{r}) \) is the electron density. Electric field we introduce as \([6,8]\] \( E_j (\mathbf{r}) = E_{0j} \exp(\varphi_i + i k_\perp y + i k_0 z) \) \((k_\perp << k_0)\). \( k_\perp \) is the wavenumber normal to the principle plane, \( k_0 \) is the wavenumber of an incident wave. Permittivity tensor \( \epsilon_j (\mathbf{r}) = \epsilon_j^{(0)} + \epsilon_j^{(1)} (\mathbf{r}) \), \( | \epsilon_j^{(1)} (\mathbf{r}) | << 1 \) contains two terms. First is a regular term, the second one is proportional to the complex phase \( \varphi_i \sim \epsilon_j^{(1)} \). Parameter \( \mu = k_\perp / k_0 \) describing diffraction effects is calculated in zero-order approximation. Diffraction effects become essential if distance travelling by wave in the magnetized plasma is big.

Fluctuation of the phase of scattered electromagnetic wave caused by electron density fluctuations satisfies the boundary condition \( \varphi_i (k_x, k_y, z = 0) = 0 \). Applying the modify perturbation method \([6,8]\] the spectral function of the phase fluctuation is:

\[
\psi(k_x, k_y, L) = A \int_0^L dz_\perp (b_3 + i b_4) n_i(k_x, k_y, z_\perp) \exp\left\{ - \gamma (b_1 + i b_2) (L - z_\perp) \right\} 
\]

(2)

where:

\[
A = (-\alpha_i + \mu \alpha_j + \Gamma \alpha_j) / \mu^2 P^2, \quad b_1 = \gamma k_\perp \left[ k_0^2 \mu P - \Gamma (k_x^2 + k_y^2 + 2 k_0 \mu k_j) \right], \quad \gamma = 1/\mu^2 P^2,
\]

\[
b_2 = \gamma \left\{ k_0^2 + \mu \left[ P (k_x + k_0 \mu) - \Gamma (k_x^2 + k_y^2 + 2 k_0 \mu k_j) \right] \right\}, \quad b_3 = \gamma k_0^2 (k_x + k_0 \mu) P, \quad b_4 = \gamma k_0^2 k_x, \quad L \text{ is a propagation distance by electromagnetic waves in the ionospheric plasma. For simplicity index } j \text{ will be withdrawn in the polarization coefficients:}
\]

\[
P_j = \frac{2 \sqrt{u (1 - v) \cos \alpha}}{u \sin^2 \alpha \pm \sqrt{u^2 \sin^4 \alpha + 4 u (1 - v) \cos^2 \alpha}}, \quad \Gamma_j = \frac{\sqrt{u \sin \alpha + P_j u \sin \alpha \cos \alpha}}{1 - u - v + u \cos^2 \alpha},
\]

upper sign and index \( j = 1 \) correspond to the extraordinary wave, lower sign and index \( j = 2 \) to the ordinary wave, \( \alpha \) is the angle between the \( \hat{Z} \)-axis (the direction of the wave propagation) and the static external magnetic field \( \mathbf{H}_0 \). These waves in magnetized plasma generally are elliptically polarized. Geomagnetic field leads to the birefringence and anisotropy.

Correlation function between two observation points spaced apart at small \( \rho_y \) and \( \rho_z \) distances in the principle and perpendicular planes, respectively, can be written in the form:

\[
V_\alpha(\rho_z, L) = 2 \pi A^2 k_0^4 L \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \left[ x^2 + P^2 (y^2 + 2 \mu y + \mu^2) \right] W_\alpha \left( x, y, -\frac{b_1}{\mu^2 P^2} \right).
\]

(3)

\[
\exp(-i \eta_z y - i \eta_y x),
\]

where

\[
b_1 = (1 + P \Gamma y + P \Gamma \mu) x^2 + P \Gamma y^3 + P (P + 3 \Gamma \mu) y^2 + P \mu (P + 2 \Gamma \mu) y, \quad \eta_z = k_0 \rho_z, \quad \eta_y = k_0 \rho_y.
\]

The double integral in the wave number space depends only on the shape of the fluctuation spectrum but not on the strength of the fluctuations. Phase fluctuations at different observation points are not independent and they correlate.
In the most interesting case of multiple scattering, when the phase fluctuations are strong \(<\phi_1^*\phi_1\gg 1\), correlation function of a scattered field in the collisionless magnetized plasma could be written as [10]:

\[
W_E(\rho_x, \rho_y, z) = E_0^2 \exp(i k_z \rho_z) \exp \left( \frac{\partial V_e}{\partial \rho_y} \rho_y + \frac{1}{2} \frac{\partial^2 V_e}{\partial \rho_y^2} \rho_y^2 + \frac{1}{2} \frac{\partial^2 V_e}{\partial \rho_y^2} \rho_y^2 \right)
\]

(4)

The derivatives of the phase correlation function are taken at the point \(\rho_x = \rho_y = 0\).

Two dimensional spatial power spectrum (SPS) of scattered radiation which is of great practical importance for, is the Fourier transformed 3D correlation function of the field [1]. This characteristic is equivalent to the ray intensity (brightness), which usually enters the radiation transport equation. At strong fluctuation of the phase \(<\phi_1^*\phi_1\gg 1\), SPS is expressed as follows:

\[
S(k_x, k_y, z) = S_0 \exp \left[ - \frac{k_x^2}{2} \frac{k_y^2}{2} \right.
\]

(5)

where: \(S_0\) is the peak value of the spectral curve, the displacement \(\Delta k_y\), the widths of the SPS in the XZ and YZ planes \(<k_y^2>\) and \(<k_x^2>\) contained derivatives of the correlation function. Further we will consider only electron density fluctuations in \(F\) region of the ionosphere.

In the ionospheric scintillation fluctuations are characterized by the scintillation index. For weak scattering of electromagnetic waves the scintillation level \(4S\) and the 2D phase spectral function describing 2D diffraction pattern at the ground are connected by the relationship:

\[
S_4^2 = 4 k_0^2 \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \ V_\phi(x, y, L) \sin^2 \left[ \Lambda_j \left(x^2 + y^2\right) \right]
\]

(6)

where: \(\Lambda_j = k_0 l / k_j\), \(k_j = (4 \pi / \lambda z)^{1/2}\) is the Fresnel wavenumber, \(z\) mean distance between the observer and plasma irregularities. The sinusoidal term is responsible for oscillations in the scintillation spectrum. The spatial autocorrelation function of the diffraction pattern could be measured with a suitable two-dimensional array of sensors.

**Numerical Calculations**

The incident electromagnetic wave has the frequency of 3 MHz propagates along the Z-axis. Plasma parameters at the altitude of 300 km are: \(u_0 = 0.22\), \(v_0 = 0.28\). An RH-560 rocket flight observations from Sriharikota rocket range (SHAR), India showed [11] that the intermediate range irregularities (100 m – 2 km) were observed in abundance in altitude regions 220-250 km and 290-320 km. Irregularities of a range of scale sizes starting from a few hundred meters to a few ten of kilometers are observed in these patches. The dip angle of the irregularities with respect to the field lines was within \(16^0\). The anisotropic spectral features in the \(F\)–region are defined for Gaussian and Power-law spectra.

New spectrum of electron density irregularities includes anisotropic Gaussian and power-law spectra [6]:

\[
W_n(k) = \frac{\sigma_n^2}{8 \pi^{5/2}} \frac{A_n l_n^3}{\chi^{2}} \left(1 + l_{\perp}^2 (k_x^2 + k_y^2) + l_{\parallel}^2 k_z^2\right)^{\mu/2} \exp \left( - \frac{k_x^2 l_{\perp}^2}{4} - p_1 \frac{k_y^2 l_{\parallel}^2}{4} - p_2 \frac{k_z^2 l_{\parallel}^2}{4} + p_3 k_y k_z l_{\parallel}^2 \right)
\]

(7)
here:  
\[ p_1 = (\sin^2 \gamma_0 + \chi^2 \cos^2 \gamma_0)^{-1} \left[ 1 + (\chi^2 - 1)^2 \sin^2 \gamma_0 \cos^2 \gamma_0 / \chi^2 \right] \]

\[ p_2 = (\sin^2 \gamma_0 + \chi^2 \cos^2 \gamma_0) / \chi^2, \]

\[ p_3 = (\chi^2 - 1) \sin \gamma_0 \cos \gamma_0 / \chi^2, \quad A_p = \Gamma \left( \frac{p}{2} \right) \Gamma \left( \frac{5-p}{2} \right) \sin \left[ \frac{(p-3)\pi}{2} \right], \quad \Gamma(x) \text{ is the gamma function,} \quad \sigma_n^2 \text{ is the variances of electron density and magnetic field fluctuations,} \quad p \text{ is the power index,} \]

\[ \chi = \frac{l_l}{l_\perp} \text{ is the anisotropy factor - the ratio of longitudinal and transverse characteristic linear sizes of plasma irregularities,} \quad \gamma_0 \text{ is the orientation angle of elongated irregularities with respect to the magnetic lines of force. The shape of electron density irregularities has a spheroidal form.} \]

Figures 1 depicts the 3D normalized correlation function of the phase fluctuations when two observation points are located in mutually perpendicular planes at distances \( \eta_x \) and \( \eta_y; \sigma_n = 10^{-3}, \)

diffraction parameter \( \mu = 0.06, \) anisotropy factor \( \chi = 10, \) plasma irregularities are field aligned \( (\gamma_0 = 0^0), \) \( \xi = 5 \) \( (l_\parallel = 800 \text{ m}). \) Figure 2 illustrates cross-sections of 3D phase correlation function of the field aligned plasma irregularities in the XZ plane for both the ordinary and extraordinary waves. The width of the curves approximately is the same in the principle YZ plane as the external magnetic field has similar influence on both waves. In the XZ plane behavior of these waves strongly differ. Broadening of the 2D correlation function for the ordinary wave 2.6 time exceeds the extraordinary

**Figure 1.** Normalized 3D correlation function of the phase fluctuations at \( \alpha = 60^0. \)

**Figure 2.** Normalized correlation function versus \( \eta_x \)

for the extraordinary (curve 1) and the ordinary (curve 2) electromagnetic waves.

**Figure 3.** Plots of the displacement of the SPS for the extraordinary wave in the YZ plane as a function of the parameter \( \chi. \)

**Figure 4.** Normalized scintillation spectrum versus parameter \( \Upsilon \)

for 3 MHz incident EM wave; \( \xi = 10. \) Curve 1 corresponds to the isotropic case \( (\chi = 1, \gamma_0 = 0^0), \) curve 2 \( (\chi = 5, \gamma_0 = 5^0), \) curve 3 \( (\chi = 12, \gamma_0 = 15^0). \)
Figure 3 depicts shift of maximum of the SPS for the extraordinary wave in the YZ plane for different orientation angle $\gamma_0$ of large-scale anisotropic plasma irregularities, $\xi=100$ ($l_l=1.6$ km). Curve 1 corresponds to the field aligned irregularities $\gamma_0=0^0$, curve 2 - $\gamma_0=10^0$, curve 3 - $\gamma_0=20^0$ and curve 4 - $\gamma_0=30^0$. Varying orientation angle in the interval $\gamma_0=0^0 \div 30^0$ displacement of the maximum increases in six times. The curves tend to the saturation at $\chi=5$ for $\gamma_0=0^0$ and at $\chi=15$ for $\gamma_0=30^0$. Figure 4 illustrates scintillation level of scattered radiation of an incident wave 3 MHz. Oscillations are observed since $\Upsilon=25$ even at $\gamma_0=0^0$ increasing parameter from $\chi=25$. Oscillations are weakly damping up to $\Upsilon=500$ and after they become stationary. The angle $\gamma_0$ has no influence on the oscillations amplitude up to $\chi\leq 25$; starting at $\chi=26$ oscillations become stationary from $\Upsilon=18$ at arbitrary $\gamma_0$. In the case $\chi=30$ and $\gamma_0=20^0$ oscillations become stationary starting from $\Upsilon=38$. Small scintillation level corresponds to $S_{4+} < 0.5$ (the positive and negative intensity fluctuations with respect to the mean level) and big scintillation level $S_{4+} > 0.5$ is caused by the positive intensity fluctuations.

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References