Theoretical Modeling for a Kind of Electric-magnetic Drive System

Yan ZHANG\textsuperscript{1}, Xiang-yu CHENG\textsuperscript{2,3,4,*} and Wen-ge CHEN\textsuperscript{2}

\textsuperscript{1}Anhui Water Conservancy Technical College, Hefei 231603, China
\textsuperscript{2}High Magnetic Field Laboratory, Chinese Academy of Sciences, Hefei 230031, China
\textsuperscript{3}University of Science and Technology of China, Hefei 230026, China
\textsuperscript{4}The 38th Research Institute of China Electronics Technology Group Corporation, Hefei 230088, China

*Corresponding author

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Abstract. AC current-carrying coil has a repulsive force produced to which a metal plate approaches. But how to calculate the force magnitude and what’s the primary cause for the repulsive force is not very clear nowadays. Besides complex existing methods, the paper provides another newer effective way to solve the problem. That’s both suitable for low-frequency and high-frequency wave coil and its nearby metal plate. It can also interpret why the electric-magnetic drive system generate force in the space. That is an enormous progress over the deceptive electric-magnetic drive system.

Introduction

Lenz’s law \cite{1} says: The direction of current induced in a conductor by a changing magnetic field due to Faraday’s law of induction will be such that it will create a magnetic field that opposes the change that produced it. But it hasn’t give us a direct quantitative relation between changing magnetic field and the conductor. Normally, we want to know how intense changing magnetic field can produce, and how high magnetic field and magnetic force can emerge between them.

It is a complex problem because of the relative parameters are plentiful including kinds of dimensions, current, materials, permeability, and so on. How to solve such a complex problem within a brief approximate method is our research goal. Based on series hypotheses, we derived the finally formula which is certified to well accord with latterly experiment result.

Hypotheses and Reasons of Repulsive Force Producing

The hypotheses are as follows:

1. Current coil’s changing magnetic field produce magnetic waves emitting from the coil. The spread speed in the surrounding space of the magnetic waves is as same as the light.

2. When the electromagnetic waves emitting form the original coil arrive at a metal induction body surface, they produce an induced electromotive force (EMF) at the metal induction body surface. EMF subjects to Faraday’s law which can be expressed in

\[ \varepsilon = -\frac{d\Phi}{dt}. \]  

3 EMF produces an induction current on the metal surface, and then the induction current produces a new EMF. So back and forth on the induction metal surface until the intensity of them gradually damp down to zero that is affected by metal self-resistance. Hence a synthetic changing induction magnetic field added from all of above induction fields is finally produced near the metal surface.

4. The final synthetic changing induction magnetic field can produce its electromagnetic waves. And the electromagnetic waves reflected by certain metal object far from the original coil will spread back to the near of original coil at speed of light.
5. The turned electromagnetic waves producing a changing induction magnetic field near the original current coil can nextly produce an effect on the original current coil, and result in a mechanical force on coil changing periodically.

6. The value of the integral changing force in a single period divided by the periodic time is the appearance force measured lastly.

On these hypotheses, we sketch an illustration for generating process of the force between a AC current-carrying coil [2] and a conductor metal as follows (refer with: Fig. 1).

![Figure 1](image-url)

Figure 1. The phase and amplitude of electromagnetic waves emitted by original coil are changed after transformation through a space and reflection back by a metal plate, which lead to the repulsive force produced.

By the sketch, the reason can be clearly traced why the repulsive force finally generate between the original coil and the inducting reflector. A simple description for the process is as follows:

When we electrify the original AC energizing coil, AC electromagnetic field waves produce near the coil. So spatial choppy electromagnetic waves generate and spread into far distant space. When the electromagnetic waves arrive at a metal plate, it causes an electromagnetic induction on the surface of the plate, and produces an induced voltage subjecting to Faraday’s law (refer with: Eq. 1). By the process of hypothesis 3, a final synthetic changing induction electromagnetic field can generate on the plate (or other forms of electric reflectors). So, electromagnetic waves produced by the induction electromagnet field can turn back to the original AC coil after spreading through a spatial distance between the two objects. Electromagnetic waves produce a newer changing magnetic field beside the original AC coil which can interact with the coil’s field to produce a periodic force on the coil.

When the spread time and the waves phase deviation on the reflector object are taken into account, we can find out the reason why there are repulsive force between the coil and the induction object. There are 3 stages occurring time delay which create phase lag deviation from the original waves. The first one is the waves spreading from the original AC coil to induction object because the distance between the objects needs time for passing electromagnetic waves. The second is reflector surface induction leading to phase lag deviation. Lastly, the turn waves going through the distance makes time delay leading to phase lag deviation. The first and last one can be easy to understand because of electromagnetic waves spreading in space needs time whatever fast speed it is. But for the second one, about phase lag by surface induction, it is more complex and we will further explain as follows.

**Phase and Amplitude Changing During Surface Induction**

When an incident electromagnetic wave $B_1$ arrives at an induction surface, it can be divided into a reflected wave vector $B_2$ and a portion component $B_3$ into the induction body (refer with: Fig. 2).
The component $B_m$ into the induction body can also be divided into a heat loss [3] portion $B_{cs}$ and a penetrating portion component $B_{1i}$.

Based on an assumption of sum of the vectors on each sides being plane symmetry on the reflector surface, we get a vector relation:

$$B_1 + B_2 = B_m. \quad (2)$$

The mirror vector into the induction body can also be divide into two parts. They satisfy

$$B_m = B_{cs} + B_{1i}. \quad (3)$$

The incident wave vector $B_1$ and penetration wave vector [4] $B_{1i}$ should have the same phase, but difference in amplitude. The heat loss vector $B_{cs}$ should perpendicular to vector $B_{1i}$:

$$B_{cs} \perp B_{1i}. \quad (4)$$

So a vector relation figure about phases and amplitudes of above mentioned vectors can be drawn out (refer with: Fig. 3).

**Quantity Relations**

According to the vector relation figure, we attempt to discuss some typical cases and critical conditions to judge if they are consistent with the actual situations which can verify the correctness of initial assumptions and their subsequent inferences.

**Case 1:** For superconductor reflector surface, the inner field

$$B_m = 0.$$ Combing Eq. 3 and Eq. 4, $B_{cs} = 0$, Penetration wave field $B_{1i} = 0$, Reflected wave field $|B_1| = |B_i|$ in the opposite direction. Those results all exactly meet the realities.

**Case 2:** For nonconductor material reflector surface like as plastic, epoxy, and so on, the inner field and penetrating portion component [5] $B_m \approx B_{1i} \approx B_i$. The heat loss vector $B_{cs} \approx 0$. Reflected wave field vector $B_2 \approx 0$. Those both also satisfy all of realities and the vector graphics.

**Case 3:** For mostly normal reflector surface, we have

$$|B_1|^2 = |B_{cs}|^2 + |B_m|^2 = |B_2|^2 + |B_{1i}|^2 \geq |B_2|^2 + |B_{1i}|^2. \quad (5)$$

Where $B_{cs}$ is the heat loss vector [6]. In case of surrounding medium permeability $\mu$, we can get the magnetic field energy converting to heat loss is

$$E_{R1} = \frac{\Delta V}{2\mu} |B_{cs}|^2 = \frac{\Delta V}{2\mu} (|B_1|^2 - |B_{1i}|^2 - |B_2|^2), \quad (6)$$

because of the reflector existing resistance.

Figure 2. Reflective process on induction surface of a plate.
Figure 3. The relationship of component vectors.

When a magnetic field wave arrived the surface of a reflector, it produces a loop current because of electromagnetic induction. Then, the loop current can also produce an induction field at near space, and the producing cycle repeats. The induction field on the surface of reflector is vector based, so they have the property of accumulating indefinitely. The lastly induction field vector is the sum of all the cycle step inductions.

For resistance existing, we assume current loop resistance

\[ r = k_p \rho. \] (7)

In the first induction, peak value of changing magnetic flux per unit time

\[ u_1 = \frac{\Delta \Phi_1}{\Delta t} = k_f f \Delta \Phi_1, \] (8)

induction current

\[ i_1 = \frac{u_1}{r} = \frac{k_f}{k_p} \Delta \Phi_1 \cdot \frac{f}{\rho}. \] (9)

In the second induction, maximum value of changing magnetic flux

\[ \Delta \Phi_2 = \Delta \beta_2 \cdot S = k_B \cdot i_1, \] (10)

peak value of changing magnetic flux per unit time

\[ u_2 = \frac{\Delta \Phi_2}{\Delta t} = k_f f \Delta \Phi_2 = k_f k_B f i_1 = k_f k_B f \frac{k_f}{k_p} \Delta \Phi_1 \cdot \frac{f}{\rho} = \frac{k_f^2 k_B}{k_p} \frac{\Delta \Phi_1}{\rho} \frac{f^2}{\rho}, \] (11)

induction current

\[ i_2 = \frac{u_2}{r} = \frac{k_f^2 k_B}{k_p} \frac{\Delta \Phi_1}{\rho} \cdot \frac{f^2}{\rho} \cdot \frac{1}{k_f k_B} = \frac{k_f}{k_p} \Delta \Phi_1 \cdot \frac{f}{\rho} \cdot \frac{k_f f}{k_p} = k_{\beta \rho} \frac{f}{\rho} \cdot i_1. \] (12)

So, we get the induction current ratio in very induction step is \( k_{\beta \rho} \frac{f}{\rho} \). The ratio of magnetic field produced by the induction current also satisfy the value \( k_{\beta \rho} \frac{f}{\rho} \). By Faraday’s law, we know them phase offset between the adjacent 2 induction steps is \( \frac{\pi}{2} \).

Hence, in all of the unlimited induction, magnetic induction field amplitudes subject to a geometric series with ratio \( K = k_{\beta \rho} \frac{f}{\rho} \), and magnetic induction field phase offset in very adjacent step is \( \frac{\pi}{2} \) (refer with: Fig. 4).
If the initial incident wave field $B_{i1} = B_i = \sin(\omega t)$, then the first induction wave field $B_{i12} = K \sin(\omega t - \frac{\pi}{2}) = -K \cos(\omega t)$. So the second induction wave field $B_{i13} = K \cdot -K \cdot \cos(\omega t - \frac{\pi}{2}) = -K^2 \sin(\omega t)$. The third induction wave field $B_{i14} = K \cdot -K^2 \cdot \sin(\omega t - \frac{\pi}{2}) = K^3 \cos(\omega t)$. The fourth induction wave field $B_{i15} = K \cdot K^3 \cdot \cos(\omega t - \frac{\pi}{2}) = K^4 \sin(\omega t)$. And so on.

Then the sum of all induction wave field is

$$B_z = B_{i12} + B_{i13} + B_{i14} + B_{i15} + \cdots$$

$$= (-K^2 + K^4 - K^6 + \cdots) \sin(\omega t) + (-K + K^3 - K^5 + \cdots) \cos(\omega t)$$

$$= \frac{-K^2}{1 + K^2} \sin(\omega t) - \frac{K}{1 + K^2} \cos(\omega t)$$

$$= \frac{K}{\sqrt{1 + K^2}} \left[ -\frac{K}{\sqrt{1 + K^2}} \sin(\omega t) - \frac{1}{\sqrt{1 + K^2}} \cos(\omega t) \right].$$

Substituting

$$\varphi = \frac{\pi}{2} + \arcsin \frac{K}{\sqrt{1 + K^2}}, \quad \cos \varphi = \frac{-K}{\sqrt{1 + K^2}}, \quad \sin \varphi = \frac{1}{\sqrt{1 + K^2}},$$

we get the reflected electromagnetic wave field [7]

$$B_z = \frac{K}{\sqrt{1 + K^2}} \sin(\omega t - \varphi),$$

where $\frac{K}{\sqrt{1 + K^2}}$ is amplitude of reflected wave field ($K = k_{\rho_0} f = \frac{k_{\rho_0}}{\lambda \rho}$), $\lambda$ is initial incident field wavelength, $f$ is supply frequency, $\rho$ is induction material electrical resistivity, $\varphi$ is reflected wave field phase.

Consider an extreme case, when the reflected body material is superconductor, its electric resistivity $\rho \to 0$, $K \to \infty$. So the amplitude is $\frac{K}{\sqrt{1 + K^2}} \to 1$ times than initial wave field. The phase offset to the initial wave field is $\varphi \to \pi$. That is to say, relative to initial wave field, the reflected wave have a reverse phase shape, and the same amplitude.
Based on Eq. 2 and its symmetrical assumption, we get the part of wave field that enter into the reflector inner:

\[
B_{in} = B_1 + B_2 = B_{111} + B_{112} + B_{113} + B_{114} + B_{115} + \cdots
\]

\[
= (1 - K^2 + K^4 - K^6 + \cdots) \sin(\alpha t) + (-K + K^3 - K^5 + \cdots) \cos(\alpha t)
\]

\[
= \frac{1}{1 + K^2} \sin(\alpha t) - \frac{K}{1 + K^2} \cos(\alpha t)
\]

\[
= \frac{1}{\sqrt{1 + K^2}} \left[ \frac{1}{\sqrt{1 + K^2}} \sin(\alpha t) - \frac{K}{\sqrt{1 + K^2}} \cos(\alpha t) \right]
\]

(15)

Substituting \( \phi_{in} = \arcsin \frac{K}{\sqrt{1 + K^2}}, \cos \phi_{in} = \frac{1}{\sqrt{1 + K^2}}, \sin \phi_{in} = \frac{K}{\sqrt{1 + K^2}} \), we get the inner wave field

\[
B_{in} = \frac{1}{\sqrt{1 + K^2}} \sin(\omega t - \phi_{in}).
\]

(16)

where \( \frac{1}{\sqrt{1 + K^2}} \) is inner wave field amplitude \( K = k_{\rho f} \frac{f}{k_{\rho f} \rho} \), \( \phi_{in} \) is inner wave phase.

The inner wave field can be further decomposed into the penetrating wave field

\[
B_{11} = \frac{1}{1 + K^2} \sin(\alpha t) \quad \text{that keeps on moving ahead, and the heat loss component vector}
\]

\[
B_{cs} = -\frac{K}{1 + K^2} \cos(\alpha t) \quad \text{that supplies the material resistive heat loss.}
\]

In case of superconductor material reflector, there are electrical resistivity \( \rho \to 0, \ K \to \infty \). So the penetrating wave field \( B_{11} \) amplitude is \( \frac{1}{1 + K^2} \to 0 \) times than initial wave field. The heat loss component vector \( B_{cs} \) amplitude is \( -\frac{K}{1 + K^2} \to 0 \) times than initial wave field. That is to say no electromagnetic wave field travel across the surface of the superconductor induction, and enter into the inside of the superconductor. Meanwhile there are no joule heat loss produced by induction current.

**Numerical Method and the Result**

By applying the property of accumulation indefinitely of surface induction field, we draw an oscillogram using MATLAB software as follows (refer with: Fig. 5).

Figure 5. The oscillogram drawn by MATLAB software

After combing some experiments and derivations, we get a semi-rational formula to calculate the force \( F_{total} \) between the coil and induction plate as follows:
\[ F_{\text{rep}} = K_T \cdot \frac{\text{deca} \cdot L^2 \cdot d \cdot \cos \left( \frac{4\pi d}{C_{\text{lip}}} \pm \text{phase} \right)}{\mu \beta d} \]

\[ = K_T \cdot \frac{K}{\sqrt{1 + K^2}} \cdot L^2 \cdot d \cdot \cos \left( \frac{4\pi d}{C_{\text{lip}}} + \frac{\pi}{2} + \arcsin \frac{K}{\sqrt{1 + K^2}} \right) \]

\[ = -K_T \cdot \frac{K}{\sqrt{1 + K^2}} \cdot L^2 \cdot d \cdot \sin \left( \frac{4\pi d}{C_{\text{lip}}} - \arcsin \frac{K}{\sqrt{1 + K^2}} \right) , \]  

where \( K_T \) is empirical coefficient,

\[ K = k_{\beta \rho} \cdot \frac{fL}{\rho d} = \frac{k_{\beta \rho} LI}{\lambda \rho d} \]  

\( \lambda \) is initial field wavelength, \( f \) is supply frequency, \( \rho \) is electrical resistivity of induction plate, \( L \) is coil inductance, \( I \) is single strand current, \( d \) is effective plate area in perpendicular direction to \( F_{\text{rep}} \), \( \mu \) is environment permeability, \( d \) is the distance between the coil and induction plate, \( C_{\text{lip}} \) is speed of light, \( \text{phase} \) is phase offset of reflected wave (that could be obtained by numerical computation).

Notice that: when \( F_{\text{rep}} \) is a positive value, it means attractive force; otherwise, when \( F_{\text{rep}} \) is a negative value, it means repulsive force.

Summary

By analyzing the internal causes, we develop a newer method to solve the calculation problem of AC current-carrying coil and metal plate reflector system. We get a semi-rational formula which may calculate the repulsive force out. But there are still some coefficients existing in the formula have not measured and it seems also like a complicated integral is hard to achieve. These problems will be our next efforts for gradually solving to total in our future researches.

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Reference


