Optimization for Realistic Real-Time Rendering
Method for Subsurface Scattering

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ABSTRACT

Monte Carlo photon tracking is used to render translucent materials such as marble, milk, skin, etc. The technique encompasses excessive amount of subsurface scattering that is computationally expensive. Faster approaches are based on approximate models derived from observed results. While such models are efficient, they tend to miss some translucency effects in the rendered results. We present an improved approximation model for real-time rendering of materials based upon computational graphs. Excellent results are obtained improving upon the previous studies.

INTRODUCTION

The exact model of subsurface scattering can be obtained using the Monte Carlo method; However, the results are not optimal since the ultimate aim is to obtain realistic rendering in real time. Therefore, precise and efficient modeling of three-dimensional translucent objects is still problematic. Light propagation in translucent materials is complicated when the light exits from the surface of the material at a point different from the incident point. The reaction between the irradiation of the photon beam and the material at irregular angles is called subsurface scattering (SSS). In previously research on subsurface scattering, a number of approximate BSSRDF based models have been proposed to resolve this problem namely, Dipole model, QD model, PBD model. Donner et al. [1] analyzed the efficient common model for modeling different materials. Hybrid Monte Carlo method based on light beam diffusion was used to optimize rendering [2]. Directional Dipole Model was developed by Frisvad et al. [3].

Physical appearance based models have been replaced by approximations in order to render realistic images. A faster rendering process and simpler coding in approximations has been introduced, which is not only efficient but also produces realistic results[4].

BACKGROUND AND RELATED WORK

BSSRDF

The function that describes the light path in materials is BSSRDF. The outgoing radiance, \( L_o \), can be calculated using the incident radiance \( L_i \), and the BSSRDF \( S_d \) is given by the following equation[5] [6]:

\[
L_o(\vec{x}_o, \vec{\omega}_o) = \int_A \int_{2\pi} S_d(\vec{x}_i, \vec{\omega}_i; \vec{x}_o, \vec{\omega}_o) L_i(\vec{x}_i, \vec{\omega}_i) (\vec{n}, \vec{\omega}_i) d\vec{\omega}_i dA(\vec{x}_i) \quad (1)
\]
To simplify the graphic computation, the BSSRDF is written as the product of one-dimensional factor called reflectance profile $R_d$, Fresnel transmission factors $F_t$, and constant $C_0$ in order to prevent the effects induced from additional intensity [7][8]:

$$S(\mathbf{x}_i, \omega_i; \mathbf{x}_o, \omega_o) = \frac{1}{\pi} F_t(\mathbf{x}_i, \omega_i) R_d(\mathbf{x}_o - \mathbf{x}_i) \frac{F_t(\mathbf{x}_o, \omega_o)}{4 C_0 (1/\mu)} \tag{2}$$

### Reflectance Profile

In terms of the boundary condition [9], reflectance profile is equal to intensity divided by the radiant flux.

$$R_d(r) = -D \frac{\Phi (\mathbf{x}_s)}{d \phi(\mathbf{x}_i)} \tag{3}$$

Diffusion dipole approximation computes the intensity by assuming a virtual light source beneath the material at one mean free path, and the resulting radiant exposure is equal to:

$$\Phi(x) = \frac{\Phi}{4 \pi D} \left( e^{-\sigma_{tr} dr} \frac{d r}{d r} - e^{-\sigma_{tr} dv} \frac{d v}{d v} \right) \tag{4}$$

Effective transport coefficient is $\sigma_{tr} = \sqrt{3} \sigma_a \sigma_t^*$. $d_r$ and $d_v$ are the distances of the sources from the points at the surface. Thus, by assuming a normal incident light beam shining on the surface of material, we can easily derive:

$$R_d(r) = z_r (1 + \sigma_{tr} d_r) e^{-\sigma_{tr} dr} + z_v (1 + \sigma_{tr} d_v) e^{-\sigma_{tr} dv} \frac{1}{4 \pi d_r^2} \tag{5}$$

where $z_r = 1/\sigma_t^*$ and $z_v = z_r + 4 AD$ are the projected distances in the direction of the normal to the surface while $z = 0$ represents the surface. Also, the equations $d_r = \sqrt{r^2 + z_r^2}$ and $d_v = \sqrt{r^2 + z_v^2}$ represent the distances accurately as the light beam is perpendicular to the surface of the material shown in Figure 1. Also, $D = 1/3 \sigma_t^*$ represents the diffusion constant, while $A = (1 + F_{dr})/(1 - F_{dr})$ represents the change in radiance exposure defined in terms of the internal reflection occurring at the surface.

![Figure 1. BSSRDF Configuration.](image)

$\alpha^*$ is reduced albedo. $\alpha^* = \sigma_s^*/\sigma_t^*$. $\sigma_s^*$ and $\sigma_t^*$ are the reduced scattering coefficients affected by the phase function, where $\sigma_s^* = \sigma_s(1 - g)$, $\sigma_t^* = \sigma_s^* + \sigma_a$. $A = (1 + F_{dr})(1 - F_{dr})$ represents the change in radiance exposure in terms of the internal reflection.
reflections off the surface. The relative index of refraction $\eta$ of the material is used to calculate $F_{d\tau}$ using the Fresnel formula:

$$F_{d\tau} \approx \begin{cases} -0.44 + 0.71/\eta - 0.33/\eta^2 + 0.06/\eta^3, & \eta < 1 \\ -1.44/\eta^2 + 0.71/\eta + 0.67 + 0.06\eta, & \eta > 1 \end{cases}$$ \hspace{1cm} (6)$$

### Oblique Incident Angles

The directional dipole diffusion is used to deal with the oblique incidence angle based on the physics. However, the inclination angle of the incident beam with the diffusion profile can cause a significant near-surface effect, which can be reduced by introducing an empirical correction factor. Donner et al. optimized the light emittance based upon the diffusion approximation [10]. By introducing an attenuation function $\kappa(x)$, we can shift the reflectance profile of the beam. By assuming that light scattering is always within the material, we have:

$$\kappa(x) = 1 - e^{-\sigma_x x} \hspace{1cm} (7)$$

The actual sources are the infinite ones along the direction of the refraction (Figure 2); hence, we can use equation 7 to calculate the integral of the spatial differential:

$$R_{r\tau}(r) = \int_0^\infty \kappa(d_r)e^{-\sigma^r x}\left[z_r(1 + \sigma_{tr} d_r) \frac{e^{-\sigma_{tr} d_r}}{4\pi d_r^3} + z_v(1 + \sigma_{tr} d_v) \frac{e^{-\sigma_{tr} d_v}}{4\pi d_v^3}\right]dx \hspace{1cm} (8)$$

where $z_r = x \cos \theta$ and, $\theta$ is the refraction angle between the refracted beam and the normal off the surface.

### Approximate Reflectance Profile

Sum of a series of Gaussians provides optimal solutions for the approximation of the reflectance profile. A good-fitting approximate curve describes a sum of two exponential functions proportional to the reciprocal distance which is shown as follows:

$$R_d = \frac{e^{-r/d} + e^{-r/3d}}{8\pi rdr} \hspace{1cm} (9)$$

where $d$ determines the scale of curves described by the Equation 9. Parameter $s$ are set to the reciprocal of $d$, and the surface albedo $A$ is defined as follows:

$$A = \int_0^\infty R_d(r) 2\pi rdr \hspace{1cm} (10)$$
which ranges from 0 to 1 because of volume scattering and absorption. Thus, we have:

\[ R_d = A s \frac{e^{-sr} + e^{-sr/3}}{8\pi r} \] (11)

**OPTIMIZED APPROXIMATE REFLECTANCE PROFILE**

Regardless of obliqueness of the incident light, reflectance profile follows the form of dipole model that contains two exponential terms as shown in equation 5. Based on the previous theories in 2.1-2.3, equation 11 can be redefined as follows:

\[ R_d = A s \frac{B_1 e^{-sr} + B_2 e^{-sr/B_3}}{8\pi r} \] (12)

The coefficients before exponential terms, \( B_1 \) and \( B_2 \) are determined by the locations of two sources and transport coefficient. The index of second exponential term, \( B_3 \), is the ratio of \( d_v \) to \( d_r \).

As the distances from any point to real source and virtual source are different, the coefficients in two exponential terms, \( B_1 \) and \( B_2 \), also have different values. The value of \( B_3 \) must, be larger than 1 because projected distance from the virtual source to the surface is larger than projected distance from the real source to the surface. On the other hand, if \( r \) is equal to zero than the distance between incoming and outgoing rays is zero at the surface and, \( B_3 \) will become maximal. Given the relative index of refraction \( \eta = 1.3 \) for whole milk, we can easily calculate the ratio of \( d_v \) to \( d_r \) which is 4.44 by assuming the same point for both incident light and the outgoing ray. In other words, the maximum value of \( B_3 \) is equal to 4.44 which is a higher result than the one in equation 11.

\[ B_3 = \sqrt{\frac{\tan^2\theta + 1}{(\tan^2\theta + 1)/B_{3m}}} \] (13)

\[ \frac{B_1}{B_2} = \frac{\tan^2\theta + 1 + (\tan^2\theta + 1)^{3/2} \sqrt{3(1-\alpha')}}{B_{3m}^2 \tan^2\theta + 1} \] (14)

Figure 3. The relationship between \( B_3 \), \( B_1/B_2 \) and \( r \) respectively for whole milk.

Assuming \( \tan\theta = r\sigma_t/B_{3m} \), where \( \theta \) is calculated using \( dr \) and \( r \) and, \( B_{3m} \) is the maximum of \( B_3 \) for any material, then we have

\[ B_3 = \sqrt{\frac{\tan^2\theta + 1}{(\tan^2\theta + 1)/B_{3m}}} \] (13)

\[ \frac{B_1}{B_2} = \frac{\tan^2\theta + 1 + (\tan^2\theta + 1)^{3/2} \sqrt{3(1-\alpha')}}{B_{3m}^2 \tan^2\theta + 1} \] (14)
The ratio of \( B_1 \) to \( B_2 \) is computed using equation 14, and the scale of \( B_1 \) and \( B_2 \) is calculated using energy conservation. The result is shown in Figure 4.

**RESULTS**

\( A = 0.20 \)

![Graph](image)

\( A = 0.50 \)

![Graph](image)

\( A = 0.80 \)

![Graph](image)

Figure 4. Fit of various reflectance profile models for surface albedos \( A = 0.2, 0.5, \) and \( 0.8 \).

According to equation 12~14, we simulate the curves for three different surface albedos, and Figure 4 shows excellent results for Monte Carlo simulation for subsurface scattering after ideal diffuse transmission. The parameters \( B_1, B_2, \) and \( B_3 \) are
respectively 0.145, 0.203, 4.2 as seen on the curve in Figure 3. The results show that our approximation is close to the Monte Carlo reference with very small energy loss.

![Figure 5. Fit of s in terms of A.](image)

The Figure 5 shows the relationship between A and s. The approximation function is as follows:

\[ s = -3.1229A^3 + 8.3943A^2 - 10.16A + 6.0302 \]  

(15)

The average relative error is approximately 1.02% from the s in the previous simulation. There is not much difference between the shape of the curve and simulated data.

**FUTURE WORKS**

A new equation is derived from previous studies on subsurface scattering without beam diffusion distribution. Future work would contain the angular term to optimize the reflectance profile near the surface. Also, the different objects may vary in terms of physical properties that result in slight changes to the parameters.

**REFERENCES**


