A Stackelberg Game Model with Supply Chain Option Contracts under Random Yield and Stochastic Demand

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Keywords: Random yield, Stochastic demand, Option contracts, Stackelberg game model.

Abstract. Consider a supply chain consisting of a retailer and a supplier, and the yield of the supplier is random while the market demand of the retailer is stochastic. Due to the double marginalization, the wholesale price contract can’t achieve the coordination of the supply chain, we introduce option contracts into the supply chain in order to investigate whether option contracts can improve the supply chain’s performance under the circumstance where the yield and demand are all uncertain. Through establishing the dynamic Stackelberg game model, which is dominant by the retailer, we discuss the optimal production strategy of the supplier and the optimal purchasing strategy of the retailer, find that the relationship between the production input and the option order quantity is non-linear. In addition, sensitivity analysis indicate that the correlations of decision variables with several parameters. Researches show that option contracts not only offering flexibility for retailer’s purchasing, it can also achieve supply chain coordination under some certain conditions.

Introduction
The chief task of the supply chain management is to balance the market demand and the production supply. In actual operation, the yield and demand of products are often uncertain. On the one hand, the production processes of many products are susceptible to many extraneous factors, which will result in the actual output differs from the production input. In many high-tech industries like electronic products, semiconductor industry, the yield is always uncertain because of the complicated production process, intricate equipment and high-quality requirements (Wang, 2009; Giri, 2011). In agricultural production, the harvest of agriculture is always out of control of the supplier, since the yield is susceptible to the weather, fertilizer and insects (Nong, 2013). On the other hand, with the intensification of market competition, the product life cycle becomes shorter. Besides, due to the influences of many factors, such as the demand and price elasticity of products, the substitutability of similar products and the level of consumer’s income, etc, the demand in the end market is also uncertain (Lariviere, 1999; Hu, 2013). Yield randomness can result in serious supply shortage, excessive production resources and decrease the market value of relevant enterprises. When retailer’s order quantity can’t satisfy market demand, there will not only occur shortage cost, shortage can also weaken the market competitiveness of enterprises (Chen & Xiao, 2015), while excessive ordering results in inventory costs (Giri, 2011). In addition, supply chain members are inclined to make their decisions to maximize their own profit, which may provide lower total profits for the integrated supply chain. Therefore, the resources in the supply chain can not be effectively configured, how to choose production strategy and ordering strategy under the scenario of random yield and stochastic demand are great challenges for supply chain managers.

Supply contracts as motivate mechanism, has drew many academician’s attention. Due to the double marginalization, the wholesale price contract can’t coordinate the supply chain (Spengler, 1950; Cachon, 2004; Li, 2013). The option contract as another coordination mechanism has been recently introduced (Spinler, 2003; Wang, 2007; Padilla, 2009; Zhao, 2010). Barnes (2002) investigated the effects of the option in supply chain coordination, and illustrated that option contract
can improve both the buyer’s profit and supplier’s revenue while it provide flexibility for the buyer to respond to uncertain demand. Different with Barnes’s study, Xu (2010) and Li (2017) assumed that both the output and demand are all uncertain, and studied the production plan of the supplier and the procurement strategy of the manufacturer through option contracts. Besides, they also considered the instantaneous purchase of the manufacturer and the emergency production of the supplier. Arani (2016) introduced option contracts and revenue-sharing contract simultaneously to the supply chain. Built game models respectively from retailer-led and manufacturer-led supply chains, and derived the optimal production input and purchasing strategies of the supply chain members. Cai (2017) introduced option contract to the vendor-managed inventory (VMI) supply chain. Under the precondition of yield uncertainty, he analised the relationship between option contract and the subsidy contract under deterministic demand. Further more, he studied option contract with replenishment tactic can coordinate supply chain and improve the performance of the supply chain under uncertain demand. Chen and Xiao (2015) introduced backup sourcing strategy into a supply chain with random yield and stochastic demand, developed a game model and analysed the value of backup contracts in the retailer dominant supply chain. However, there is still a small amount of research on random yield and stochastic demand with option contracts.

In this paper, we analyze the value of option contracts in a two-echelon supply chain, different with Hamed’s study, in which the market demand and retail price of products are assumed to be stochastic, the yield and the market demand are considered both random in our paper. Similar to Wang and Gerchak’s (1996) study, we use multiplicative form to characterize the uncertainty of yield. Through establishing the Stackelberg game model, which is dominant by the retailer, we discuss the optimal production strategy of the supplier and the optimal purchasing strategy of the retailer, and we also analyze the performance of the supply chain.

The Stackelberg Model with Supply Chain Option Contracts

Model Formulation and Assumption

As the supply chain is dominant by the retailer, the event sequence under the Stackelberg model can be described as follows: firstly, the retailer chooses the order quantity of options, \( q \), with a unit option price \( o \). Secondly, according to retailer’s option order quantity, the supplier decides the production input, \( Q \), with a unit production cost \( c \), and the actual output of products is \( \epsilon Q \). Thirdly, at the beginning of the selling season, the retailer determines the exercise quantity of the option with a unit exercise price \( e \). When the actual yield is less than the quantity of executing option, the supplier needs to pay penalty to retailer for the inadequate part with a unit penalty cost \( \beta \). When the actual yield exceeds the order quantity of retailer, the retailer can take instantaneous purchase from the supplier when necessary, with a unit instantaneous order cost \( \theta \). A shortage cost \( b \) may be incurred when the retailer can’t satisfy the market demand. At the end of the sales period, the extra products of the retailer or supplier will have a salvage value \( v \).

Let \( p \) be the retail price of the retailer; \( x \) as a stochastic variable denotes the market demand, and \( x \in (0, \infty) \), \( \mu \) is the mean value of \( x \), \( F(x) \) and \( f(x) \) denote cumulative distribution function and probability density function of the stochastic variable respectively; \( \epsilon \) is a random factor, and \( \epsilon \in (0, 1) \), \( \delta \) is the mean value of \( \epsilon \), \( G(\epsilon) \) and \( g(\epsilon) \) denote cumulative distribution function and probability density function of the random factor respectively. We assume that both the retailer and the supplier are risk neutral; \( v < \frac{\delta}{\delta} < o + e < o + \theta < p \), \( e + \beta > \theta \), \( p + b < e + \beta \), which ensure that both the retailer and the supplier can obtain profit and avoid trivial and unreasonable cases.
The Centralised Supply Chain

Since the strategy of the centralised supply chain is regarded as a benchmark of the decentralized, we study the production strategy of the centralised system in this section. The profit function of the integrated supply chain is given by

$$\pi_{sc} = p \min\{x, cQ_{sc}\} - cQ_{sc} - b \max\{(x - cQ_{sc}), 0\} + v \max\{(eQ_{sc} - x), 0\}.$$  

The first and the second terms represent the total revenue and the total production costs of the channel respectively; the third term is the shortage cost of the supply chain when the actual yield is less than the market demand; the last term is the salvage value when the productions surpass the market demand. The expected profit of the supply chain can be simplified as follows:

$$E(\pi_{sc}) = \int_0^1 \left[ (p + b - v)(x - eQ_{sc}) f(x) g(e) dx \right. \left. + \mu \delta - cQ_{sc} + (p + b)\delta Q_{sc} \right].$$

**Proposition 1:** The optimal production input $Q_{sc}^*$ of the centralised supply chain satisfies the following equation:

$$\int_0^1 e[1 - F(eQ_{sc}^*)]g(e)de = \frac{c - v\delta}{p + b - v}.$$  

The Decentralised Supply Chain

As the retailer is the leader of Stackelberg game model, we solve the optimal production input strategy of the supplier firstly according to the backward induction principle. The supplier’s utility function can be expressed as follows:

$$\pi_s = oq + e \min\{x, q, eQ\} - cQ - \beta \min\{(x - eQ)^+, (q - eQ)^+\} + \theta \min\{(x - q)^+, (eQ - q)^+\} + v(eQ - x)^+.$$  

The first term is the revenue of selling $q$ unit options; the second term is revenue of executing options from the retailer; the third is the costs of production input; the fourth is the penalty paid to the retailer when the actual yield is less than the quantity of executing options; the fifth is revenue when retailer adopts instantaneous purchase; the last is the salvage value. The expected profit of the supplier can be simplified as follows:

$$E(\pi_s) = \int_0^q \left[ e + \beta - \theta(x - eQ) f(x) g(e) dx \right. \left. + \mu \delta - cQ + (p + b)\delta Q \right].$$

**Proposition 2:** The optimal production input $Q_s^*$ of the centralised supply chain satisfies the following equation, and $Q_s^*(q)$ is the reaction function in $q$.

$$\int_0^q (e + \beta - \theta)(x - Q) f(x) dx + \int_0^q (v - \theta)(eQ - x) f(x) g(e) dx \delta - (o + e - \theta)q + (\theta\delta - c)Q$$

It can be found that the optimal production strategy depends on several parameters, such as the exercising price of option, unit production cost and unit penalty cost. According to the implicit function theorem of equation (6), we can derive:
\[
\frac{dQ^*(q)}{dq} = \frac{A}{B}.
\]

In equation (7), the capital letter A and B represent equation (8) and (9) respectively,

\[
A = (e + \beta - \theta) \overline{F}(q) \frac{q}{Q'(q)} g\left(\frac{q}{Q'(q)}\right).
\]

\[
B = (e + \beta - \theta) \overline{F}(q) \left(\frac{q}{Q'(q)}\right)^2 g\left(\frac{q}{Q'(q)}\right)
\]

\[+ Q'(q) \int_{0}^{q} (e + \beta - \theta) f(\varepsilon Q^*(q)) \varepsilon^2 g(\varepsilon) d\varepsilon - Q^*(q) \int_{0}^{1} (v - \theta) f(\varepsilon Q^*(q)) \varepsilon^2 g(\varepsilon) d\varepsilon.
\]

From equation (6) and (7), we can observe that the optimal production input of the supplier is response to the option order quantity of the retailer, but the relationship between the production input and the option order quantity is non-linear.

Then, the retailer chooses the optimal option order quantity according to the reaction function. The retailer’s profit function can be denoted as follows:

\[
\pi_r = p \min\{\varepsilon Q^*(q), x\} - oq - e \min\{\varepsilon Q^*(q), q, x\}
+ \beta \min\{(x - \varepsilon Q^*(q))^+, (q - \varepsilon Q^*(q))^+\}
- \theta \min\{(x - q)^+, (\varepsilon Q^*(q) - q)^+\} - b(x - \varepsilon Q^*(q))^+
\]

The first term is the total sales income of the retailer, the second and third denote the costs of purchasing and exercising options respectively, the fourth is the penalty revenue from the supplier when the supplier’s actual yield is less than the quantity of executing options; the fifth is instantaneous purchase costs when the retailer adopts instantaneous purchase from the supplier, the last is the shortage cost of the retailer when the retailer can’t satisfy the final market demand. The expected profit of the retailer can be simplified as follows:

\[
E(\pi_r) = \int_{0}^{Q^*(q)} \int_{0}^{q} (e + \beta - \theta)(x - q) f(x) g(\varepsilon) dx d\varepsilon
- \int_{0}^{Q^*(q)} \int_{0}^{\varepsilon Q^*(q)} (e + \beta - \theta)(x - \varepsilon Q^*(q)) f(x) g(\varepsilon) dx d\varepsilon
- \int_{0}^{Q^*(q)} (e + \beta - \theta)(\varepsilon Q^*(q) - q) g(\varepsilon) d\varepsilon - \int_{0}^{1} (e - \theta)(x - q) f(x) dx.
\]

**Proposition 3:** The optimal option order quantity \( q^* \) of the retailer satisfies the following equation:

\[
\int_{0}^{q^*} (e + \beta - \theta) \overline{F}(q^*) g(\varepsilon) d\varepsilon - \int_{0}^{q^*} (e + \beta - \theta) \frac{dQ^*(q^*)}{dq} \bigg|_{q=q^*} \overline{F}(\varepsilon Q^*(q^*)) \varepsilon^2 g(\varepsilon) d\varepsilon
- \int_{0}^{1} (p + b - \theta) \frac{dQ^*(q^*)}{dq} \bigg|_{q=q^*} F(\varepsilon Q^*(q^*)) \varepsilon g(\varepsilon) d\varepsilon
+ (e - \theta) F(q^*) + (p + b - \theta) \delta \frac{dQ^*(q^*)}{dq} \bigg|_{q=q^*} = o + e - \theta
\]

We can also find that the optimal solution \( q^* \) depends on parameters: \( o, e, p, b, \theta, \beta \), and it will changes along with any one of those parameters.

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Supply Chain Coordination

Compared the profit of the centralised supply chain with the total profits of the decentralized, we find that the supply chain can achieve coordination when \( Q^* = Q_{sc}^* \). Let \( Q^* = Q_{sc}^* \), we can derive the following equation:

\[
\int_0^Q \varepsilon [1 - F(\varepsilon Q_{sc}^*)] g(\varepsilon) d\varepsilon = \frac{p + b - \theta}{e + \beta - \theta} \int_0^1 \varepsilon [1 - F(\varepsilon Q_{sc}^*)] g(\varepsilon) d\varepsilon .
\]  

(13)

And the condition on which the supply chain achieves coordination can be derived from solving the above equation. In other words, when the parameters satisfy the equation(13), the channel will reach the Pareto optimality.

Application

The decision model in this paper can be used in those areas where both the actual production and the market demand of products are uncertain, such as electronic product, semiconductor and agricultural product, etc. And this work will provide theoretical support and decision-making reference for managers.

Conclusion

Through establishing the dynamic Stackelberg game model, which is dominant by the retailer, we discuss the optimal production strategy of the supplier and the optimal purchasing strategy of the retailer, find that the relationship between the production input and the option order quantity is non-linear. In addition, sensitivity analysis indicate that the correlations of decision variables with several parameters. Researches show that option contracts not only offering flexibility for retailer’s purchasing, it can also achieve supply chain coordination under some certain conditions. Sensitivity analysis between stochastic variables and parameters of the option contracts will provide some managerial insights for the managers. They can make the supply chain coordinated by setting the contract parameters reasonably.

Acknowledgement

This work was funded by the National Natural Science Foundation of China (71371169).

References


