Robust Controller Design for Uncertain Lure Systems
Guaranteeing Dichotomy Behaviors

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Abstract. This paper focuses on the robust controller design problem of Lure systems with parameter uncertainties. Using matrix theory, the sufficient condition for the dichotomy of uncertain Lure systems is first presented. Furthermore, the methods of designing a state feedback controller and a dynamic output feedback controller for uncertain Lure systems guaranteeing dichotomy are proposed separately. Finally a numerical example is included to show the effectiveness of the proposed approach.

Introduction

The global behaviors of nonlinear systems with multiple equilibria have attracted more and more attention. The analysis and synthesis problem of such systems are much different from systems with single equilibrium. The nonlinear systems have various global behaviors such as Lagrange stability [2-4], dichotomy [5-12], gradient-like [13-16], Bakaev stability and so on.

The frequency-domain inequality condition for the dichotomy behavior of a class of nonlinear systems was obtained in Leonov et al. (1996) [1]. A state feedback controller is designed to eliminate the chaotic phenomena by guaranteeing the chaos system dichotomy [5]. Gradient-like and dichotomy behaviors of pendulum-like dynamical networks systems are considered in [6]. The method of controller design for pendulum-like system guaranteeing dichotomy is proposed by KYP lemma and positive real control in [7]. Mixed real/complex dynamic uncertainties are concerned and the condition of robust dichotomy of the Lure system is obtained in [8]. In [9], a dynamic output feedback controller is designed for Lure systems guaranteeing dichotomy which is less conservative than previous papers. The effects of interconnections are discussed in [11-12]. However the uncertainties are not considered in [7] and [9], which cannot be avoided in practice. Although uncertainties are concerned in [8], the dichotomy controller is not studied.

Based on the mentioned above, this paper is devoted to designing controllers with less conservativeness for uncertain Lure systems. A LMI-based method for dichotomy analysis of the uncertain Lure system is first presented. And a state feedback controller and a dynamic output feedback controller are designed separately for the uncertain Lure system guaranteeing dichotomy.

Preliminaries and Problem Statement

Consider the following Lure system

\[ \dot{x} = Ax + B\varphi(\sigma), \quad \dot{\sigma} = Cx + D\varphi(\sigma), \]  

where \( A \in \mathbb{R}^{n \times n} \) has no eigenvalues on the imaginary axis, \( B, C^T \in \mathbb{R}^{n \times n} \), \( D \in \mathbb{R}^{n} \), and the nonlinear function \( \varphi : \mathbb{R} \to \mathbb{R} \) is piecewise continuously differentiable. \( \varphi(\sigma) \) exists for all \( \sigma \in \mathbb{R} \), and there exist two numbers \( \mu_1 \) and \( \mu_2 \) such that

\[ -\infty < \mu_1 \leq \frac{d\varphi}{d\sigma} \leq \mu_2 < +\infty, \forall \sigma \in \mathbb{R}. \]  

(2)
The transfer function of the linear part of (1) from the input \( \varphi \) to the output \( -\dot{\sigma} \) is

\[
G(s) = C(A - sI)^{-1}B - D.
\]

Lemma 2.1 [8] Suppose \((A, B)\) is controllable, \((A, C)\) is observable, and \(G(s) \in RL_{\infty}\). System (1) is quasi-dichotomous, if there exist a matrix \( P = P^T \) and numbers \( \gamma, \chi, \varepsilon > 0, \tau > 0 \) such that

\[
\begin{bmatrix}
\text{He}(PA) + \varepsilon C^T C + \varepsilon D^T D + \eta \lambda E_i^T E_i & \frac{\chi}{2} C^T E_i & \alpha C^T E_i \\
\frac{\chi}{2} He(D) + \varepsilon D^T E_i & \gamma + \alpha D^T \\
\end{bmatrix} < 0,
\]

where \( \varepsilon = \varepsilon - \tau \mu_1, \mu_2 \) and \( \alpha = \tau \mu_1 + \mu_2 \).

Lemma 2.2 [16] If all requirements of Lemma 2.1 are true and the matrix \( A \) is Hurwitzian, then system (1) is dichotomous.

Lemma 2.3 [16] If all requirements of Lemma 2.1 are true and \( \varphi(\sigma) \) has a finite number of isolated zeros, then system (1) is dichotomous.

Lemma 2.3 [17] For any constant matrices \( T_1, T_2, \) a symmetric matrix \( Q \) and a matrix \( F \) satisfies \( F^T F \leq \lambda^2 I \). Then \( Q + \text{He}(T_1 F T_2) < 0 \) holds if and only if \( Q + \eta \lambda^2 T_1^T T_1^T + \eta^{-1} T_2^T T_2 < 0, \eta > 0 \).

Main Results

In this section, the sufficient condition for uncertain Lure system being dichotomous is presented. And the methods of designing a state feedback controller and a dynamic output feedback controller guaranteeing the closed-loop system dichotomy behaviors are proposed respectively.

Robust Dichotomy Analysis

Consider the following uncertain Lure system

\[
\dot{x} = (A + \Delta A)x + (B + \Delta B)\varphi(\sigma), \quad \dot{\sigma} = Cx + D\varphi(\sigma),
\]

where \( A, B, C, D \) and \( \varphi \) are defined as in Section 2, and \( \Delta A, \Delta B \) have the form of

\[
(\Delta A \quad \Delta B) = HF \begin{pmatrix} E_1 & E_2 \end{pmatrix},
\]

where \( H \in R^{n \times i}, E_1 \in R^{j \times n}, E_2 \in R^{j \times d} \) are known constant matrices, and \( F \in R^{n \times j} \) is an unknown matrix satisfying \( F^T F \leq \lambda^2 I \) with \( \lambda > 0 \) is a known constant.

Theorem 3.1 Suppose \((A, B)\) is controllable, \((A, C)\) is observable, and \(G(s) \in RL_{\infty}\). System (4) is quasi-dichotomous, if there exist a matrix \( P = P^T \) and numbers \( \gamma, \chi, \varepsilon > 0, \tau > 0, \eta > 0 \) such that

\[
\begin{bmatrix}
\text{He}(PA) + \varepsilon C^T C + \eta \lambda E_i^T E_i & \frac{\chi}{2} C^T E_i & \alpha C^T E_i & PH \\
\frac{\chi}{2} He(D) + \varepsilon D^T E_i & \gamma + \alpha D^T & 0 \\
\end{bmatrix} < 0,
\]

where \( \text{He}(A) = A^T + A \).
\begin{align*}
\text{Proof} \quad &\text{Denote } M = \begin{pmatrix}
He(PA) + \varepsilon_i C^T C & P B + \frac{\chi}{2} C^T + \varepsilon_i C^T D & \alpha C^T \\
* & -\tau & \\
* & * & -\tau
\end{pmatrix}. \text{ By Lemma 2.1, system (4) quasi-dichotomous if exist a matrix } P = P^T \text{ and numbers } \gamma, \chi, \varepsilon > 0, \tau > 0 \text{ such that }
\begin{pmatrix}
He(P\Delta A) & P\Delta B & 0 \\
* & 0 & 0 \\
* & * & 0
\end{pmatrix} < 0. \tag{7}
\end{align*}

Let \( T_i = (PH \ 0 \ 0)^T \), \( T_2 = (E_1 \ E_2 \ 0) \). Inequality (7) can be rewritten as \( M + He(T_i FT_i) < 0 \). By Lemma 2.3 and Schur complement, the above inequality holds if and only if there is a positive number \( \eta > 0 \) such that
\[
\begin{pmatrix}
M + \eta^2 T_2^T T_2 & T_i \\
T_i^T & -\eta I
\end{pmatrix} < 0.
\]

From the above inequality we get (6) and the proof is completed.

Remark 3.1 In [5], the frequency domain inequality is
\[
\begin{pmatrix}
B^T (j\omega - A)^{-1} & \varepsilon_i C^T C & \varepsilon_i DC + \frac{1}{2} \chi C - \alpha A^T C \\
* & \varepsilon_i D^2 - \tau (\mu_i + \mu_2)D^T B - \tau \omega^2 & I
\end{pmatrix} < 0,
\]
and \( \tau \omega^2 \) is supposed to be zero, which is more conservative than Lemma 2.1. \( \tau \) is set to be zero in [6], and \( \tau \) is set to be identity matrix, and \( \gamma = 0 \) in [8]. We can see that each of the conditions obtained above is just one of the situations for \( \tau \) or \( \gamma \). Theorem 3.1 is obtained directly from Lemma 2.1, which is less conservative than the sufficient conditions in [5, 6, 8].

Corollary 3.1 If all requirements of theorem 3.1 are true and matrix \( A \) is Hurwitzian or \( \varphi(\sigma) \) has a finite number of isolated zeros, system (4) is dichotomous.

Proof Corollary 3.3 can be obtained directly from Lemma 2.2 and Lemma 2.3.

Controller Design

Based on the above results, the methods of designing a state feedback controller and a dynamic output feedback controller for system (4) guaranteeing dichotomy are proposed.

Consider the following Lure system
\[
\dot{x} = (A + \Delta A)x + (B_1 + \Delta B)\varphi(\sigma) + B_2 u, \quad \dot{\sigma} = Cx + D\varphi(\sigma), \tag{8}
\]
where \( A, C, D \) and \( \varphi \) are defined as in Section 2, \( B_i \) is the same as \( B \) in system (1), \( u \in R \) is the control input and \( \Delta A, \Delta B \) satisfying (5). The state feedback controller has the form \( u = Kx \).

The closed-loop system is
\[
\dot{x} = (A + \Delta A + B_2 K)x + (B_1 + \Delta B)\varphi(\sigma), \quad \dot{\sigma} = Cx + D\varphi(\sigma). \tag{9}
\]

Theorem 3.2 The closed-loop system (9) is quasi-dichotomous with respect to (5), if there exist numbers \( \gamma, \chi, \varepsilon > 0, \tau > 0, \eta > 0 \), matrices \( B_i \) has a full column rank, \( \tilde{P} = \tilde{P}^T \) and \( \tilde{P}_{ii} \) is invertible such that

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\[
\begin{align*}
\begin{bmatrix}
He(PA + T_B^2 \bar{Y}) + \varepsilon C^T C + \eta \lambda^2 E_1 E_1 & PB_1 + \frac{1}{2} C^T + \varepsilon C^T D + \eta \lambda^2 E_1^2 E_2 & \alpha C^T & PH \\
* & \frac{1}{2} He(D) + \varepsilon D^T D + \eta \lambda^2 E_2^2 E_2 & \gamma + \alpha D^T & 0 \\
* & * & -\tau & 0 \\
* & * & * & -\eta I
\end{bmatrix} < 0,
\end{align*}
\]

satisfies, where \( P = T_B^2 \tilde{P} T_B \), \( Y = (Y_1, 0)^T \), \( Y_1 = \tilde{P}_1 K \), \( \tilde{P} = \begin{pmatrix} \tilde{P}_1 & 0 \\ 0 & \tilde{P}_{22} \end{pmatrix} \), \( T_B = \begin{pmatrix} (B_1^2 B_2)^{-1} & B_1^T \\ B_2^T & 0 \end{pmatrix} \). And the gain matrix \( K = \tilde{P}_1^{-1} Y_1 \).

**Proof** By theorem 3.1, the closed-loop system (9) with respect to (5) is quasi-dichotomous if there exist a matrix \( P = P^T \) and numbers \( \gamma, \varepsilon, \tau, \eta > 0 \) such that

\[
\begin{align*}
\begin{bmatrix}
He(P(A + B_2 K)) + \varepsilon C^T C + \eta \lambda^2 E_1 E_1 & PB_2 + \frac{1}{2} C^T + \varepsilon C^T D + \eta \lambda^2 E_1^2 E_2 & \alpha C^T & PH \\
* & \frac{1}{2} (D + D^T) + \varepsilon D^T D + \eta \lambda^2 E_2^2 E_2 & \gamma + \alpha D^T & 0 \\
* & * & -\tau & 0 \\
* & * & * & -\eta I
\end{bmatrix} < 0.
\end{align*}
\]

Let \( P = T_B^T \tilde{P} T_B \), we have \( PB_2 K = T_B^T \tilde{P} T_B K = T_B^T \left( \tilde{P}_1 K \right)^T \). From the above equality and (11) we have inequality (10). By \( Y = (Y_1, 0)^T \) and \( Y_1 = \tilde{P}_1 K \), we have \( K = \tilde{P}_1^{-1} Y_1 \). Theorem 3.2 is proved.

**Corollary 3.2** If all requirements of theorem 3.2 are true and matrix \( A \) is Hurwitzian or \( \varphi(\sigma) \) has a finite number of isolated zeros, system (9) is dichotomous.

**Proof** Corollary 3.2 can be obtained directly from Lemma 2.2 and Lemma 2.3.

Next, the dynamic output feedback controller is considered. The uncertain Lure system is

\[
\begin{align*}
\dot{x} &= (A + \Delta A)x + (B_1 + \Delta B_1)\varphi(\sigma) + B_2 u, \\
\dot{\sigma} &= C_1 x + D_1 \varphi(\sigma), \quad y = C_2 x + D_2 \varphi(\sigma),
\end{align*}
\]

where \( A, \varphi \) and \( u \) are defined as in Section 2, \( B_1, C_1, D_1 \) are the same as \( B, C, D \) in system (1), matrices \( B_2, C_2, D_{21} \) have appropriate dimensions respectively and \( y \in \mathbb{R} \) is the measured output.

The dynamic output feedback controller has the form of

\[
\dot{x}_k = A_k x_k + B_k y, \quad u = C_k x_k + D_k y.
\]

The closed-loop system is

\[
\dot{x}_c = A_c x_c + B_c \varphi(\sigma), \quad \dot{\sigma} = C_c x_c + D_c \varphi(\sigma),
\]

where

\[
x_c = \begin{pmatrix} x \\ x_k \end{pmatrix}, \quad A_c = \begin{pmatrix} A & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} \tilde{C} + \Delta \tilde{A} + \Delta B + \Delta \tilde{C} & \tilde{B} + \Delta \tilde{B} \\ \tilde{C} & \tilde{D}_{21} \end{pmatrix}, \quad J_k = \begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix}, \quad \tilde{D}_{21} = \begin{pmatrix} 0 & D_{21} \end{pmatrix}^T.
\]

\[
\tilde{A} + \Delta \tilde{A} = \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \Delta A & 0 \\ 0 & 0 \end{pmatrix}, \quad \tilde{C} = \begin{pmatrix} C_1 & 0 \end{pmatrix}, \quad \tilde{B} + \Delta \tilde{B} = \begin{pmatrix} B_1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \Delta B & 0 \\ 0 & 0 \end{pmatrix}, \quad \tilde{C} = \begin{pmatrix} 0 & I \\ 0 & C_2 \end{pmatrix}.
\]
Theorem 3.3 The closed-loop system (14) is quasi-dichotomous with respect to (5), if there exist numbers $\gamma, \chi, \varepsilon > 0, \tau > 0, \eta > 0, B_i$ has a full column rank, $\tilde{P}_{ci} = \tilde{P}_{ci}^r$ and $\tilde{P}_{cl2}$ is invertible such that

$$
\begin{bmatrix}
He(P_{ci}\tilde{A} + T_b^r Y\tilde{C}) + \varepsilon_i\tilde{C}^T\tilde{C} + \eta\lambda^2\tilde{E}_i^T\tilde{E}_i & P_{ci}\tilde{B} + T_b^r Y\tilde{D}_{21} + \frac{\varepsilon_i}{2}\tilde{C}^T + \varepsilon_i\tilde{C}^T\tilde{D} + \eta\lambda^2\tilde{E}_i^T E_2 & \alpha\tilde{C}^T & PH
\end{bmatrix}
$$

\begin{bmatrix}
\gamma & 0 \\
\tau & 0 \\
-\eta & 0
\end{bmatrix}
< 0 \tag{15}
$$

satisfies, where $\tilde{P}_{ci} = \begin{pmatrix} \tilde{P}_{ci11} & 0 \\ 0 & \tilde{P}_{ci22} \end{pmatrix}$, $T_b = \begin{pmatrix} (\tilde{B}^T\tilde{B})^{-1}\tilde{B}^T \tilde{B} \end{pmatrix}$. And the gain matrix $J_k = \tilde{P}_{ci11}^{-1}Y_2$.

Proof Let $\varepsilon > 0$, for system (14), Theorem 3.1 can be rewritten as

$$
\begin{bmatrix}
\Lambda & P_{ci}(\tilde{B} + \tilde{B}J_k\tilde{C}) + \frac{\varepsilon_i}{2}\tilde{C}^T + \varepsilon_i\tilde{C}^T\tilde{D} + \eta\lambda^2\tilde{E}_i^T E_2 & \alpha\tilde{C}^T & PH
\end{bmatrix}
$$

\begin{bmatrix}
\gamma & 0 \\
\tau & 0 \\
-\eta & 0
\end{bmatrix}
< 0 , \tag{16}
$$

where $\Lambda = He(P_{ci}(\tilde{A} + \tilde{B}J_k\tilde{C})) + \varepsilon_i\tilde{C}^T\tilde{C} + \eta\lambda^2\tilde{E}_i^T\tilde{E}_i \cdot P_{ci}\tilde{B}J_k = T_b^r \tilde{P}_{ci}T_b\tilde{B}J_k = T_b^r \left( \tilde{P}_{ci11}J_k \right)^T$ with $P_{ci} = T_b^r \tilde{P}T_b^r$. From the above equality and (16), we can have inequality (15). And by $Y = (Y_2 \ 0)^T$ and $Y_2 = \tilde{P}_{ci11}J_k$, Theorem 3.3 is proved.

Corollary 3.3 If all requirements of theorem 3.3 are true and matrix $A_{ci}$ is Hurwitzian or $\phi(\sigma)$ has a finite number of isolated zeros, system (14) is dichotomous.

Numerical Example

Consider the Lure system (1) with parameters $A_{-5 \ 3; 0 \ -0.1}, B_{-1.2 \ 1}^T, C_{2 \ -1}, D_{-0.4}$ and the nonlinear function is $\phi(\sigma) = \sin(\sigma) - 0.2$. It is obvious that $\mu_1 = 1$.

With initial value (2.5 4), the states of the above Lure system are shown in Fig.1, which show that this system is not dichotomous.

The other parameters of the uncertain Lure system (9) are $B_2 = (1 \ 2)^T, C_2 = (1 \ -1), D_{21} = 1, E_1 = (0.1 \ 0.2), E_2 = 0.1, H = (0.5 \ 1)^T$. By theorem 3.2 and corollary 3.2, the parameters obtained for state feedback controller are $P = (92.1845 \ 0; 0 92.1845), Y = (-537.6581 \ -297.2788 \ 0 0), K = (-5.8324 \ -3.2248)$. With initial value (2.5 4), the states of the Lure system with the state feedback controller are shown in Fig.2. And we can find that the closed-loop system is dichotomous.

And the parameters obtained for output feedback controller are $P = (-123.2588 \ 0 0; 6.8631 \ 0 0 \ 23.0405), Y = (-61.6294 \ 0; 61.1864 \ 0 0), K = (-0.5000 \ 0 8.9153)$. With initial value (2.5 4), the states of the Lure system with the dynamic output feedback controller are shown in Fig.3. And we can find the closed-loop system is dichotomous.
Conclusion

In this paper, the sufficient conditions guaranteeing dichotomy for uncertain Lure systems based the linear matrix inequalities are proposed. Then a state feedback controller and a dynamic output feedback controller are designed separately to guarantee the dichotomy of the uncertain Lure system such that there are no oscillations or chaotic. Finally a numerical example shows the effectiveness of the proposed method.

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References


