Dynamical Modelling and Positioning Control Simulation of a Spherical Robot Driven by Three Omnidirectional Wheels

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Abstract. In this paper, we concerned on the dynamical modelling and the position control problem for a novel spherical robot which was driven by three omnidirectional wheels. We first introduce the mechanism of our spherical robot. And then considering the noholonomic constraints of the shell contacting with the omnidirectional wheels and the shell contacting with the ground, we developed a nonlinear dynamical model for the system by use of Chaplygin equation. With the model, we suggested that the robot was an under-actuated system which consists of six independent velocities and three driving-torque inputs. By linearized the position variables of the shell, we designed a controller to regulate the position of the system according to the Theorem of Partial Feedback Linearization. Numerical simulations were exploited to show the effectiveness of our controller.

Introduction

Driving mechanism is one of the most critical factors in determining the performance of a spherical robot. In the past four decades, several kinds of mechanism have been used to design a spherical robot. Considering the working principle differences of the driving mechanism, at present, the spherical robot can be sorted into two types, that is, the eccentric moment driving spherical robot and the angular momentum driving spherical robot.

For the former, the working principle was summarized as that the driving mechanism of the spherical robot force the COG (center of gravity) of the system deviating from the vertical sphere center line, in this case, the balance of the system is destroyed, and under the action of gravity moment the sphere shell then run on the ground. In [1], A. Halme et al. proposed a spherical robot with a single wheel on the bottom of the spherical shell. He believed when the wheel rolls along the inner surface of the shell, an eccentric moment would be aroused, so the shell began running at once. In [2], Ranjan Mukhejee et al. presented a spherical robot that was driven by four moving masses. Mukhejee gave the idea when the masses move on their individual leadscrew along the sphere diameter, the COM (center of mass) of the system should be changed, and as a result, the shell would begin to roll. In [3,4], There are similar scheme. In [5], J Alves et al. considered a four wheels vehicle in his spherical robot. Alves suggested that the running vehicle on the bottom of the inner shell could give a moment to drive the shell running. In [6], Zhan et al. exploited an omnidirectional as steering wheel and a longitudinal driving wheel to drive the sphere shell. Zhan addressed that if the exact rotations of the two wheels were perform, the shell should run in all direction. Similar to Zhan, in [7], Wang discussed a spherical robot of two omnidirectional wheels on two orthogonally axis. In [8], Sun et al. proposed a weight-pendulum of two degree of freedom in their spherical robot. Sun argued that, as the pendulum could be driven to swing in all directions, the robot then might run in omnidirection. In [9], Zhao et al. presented a spherical robot with two coaxial eccentric masses. Zhao believed when the masses run with different acceleration and velocity, the robot should consequently move in all direction. The similar mechanism was also proposed in [10].

For the latter, the working principle was as follow: the spherical robot usually configures with a high speed rotating rotor inside the sphere shell. If we ignore the external friction torque between
the running shell and the ground, with the principle of the angular momentum conservation, the shell would rotate reverse to that of the running rotor. Furthermore, if we alternated the speed and direction of the rotating rotor, the omnidirectional motion of the spherical robot then could be gotten. In [11], Toshiaki Otani et al. developed a spherical robot driven by a three axis mechanical gyroscope. The robot’s gyroscope was fixed on a gimbals inside the sphere, and when it rotated with high speed, they believed, there would be a counter moment that can drive the shell to run definitely. In [12], V. Joshi et al. proposed a spherical robot with double rotors which shared the similar principle in [11].

By now, there is a trend that the spherical robot of the former showed more advantage than the spherical robot of the latter, e.g., more steering flexibility, lower speed of the driving mechanism. Especially, the robot developed recently in [9] and [10] that used omnidirectional wheels, have exhibited a great deal of extinguished performance. For this reason, the former is still attracting in spherical robot research. Furthermore, utilizing the omnidirectional wheels to develop a driving mechanism for a spherical robot might be a promising technology in the future.

In this paper, we consider a new type of spherical robot driven by three inner omnidirectional wheels. At first, we introduce the mechanical structure of our spherical robot in detail; and next, we develop a dynamical model for the robot system by using Chaplygin equation; moreover, we design a position regulating controller by the Theorem of Partial feedback Linearization for the system and perform a simulation to testify the effectiveness of our derivation.

Mechanism and Working Principle

The spherical robot in [9] and [10] were designed with one wheel contacting with their shell, which might restrict the driving capability of the system, thus we use three omnidirectional wheels to improve our robot’s performance.

Mechanical Structure

The structure of the system is shown in Figure 1~Figure 2.

![Figure 1. Principle prototype of our spherical robot.](image1)

![Figure 2. The schematic diagram of our spherical robot.](image2)

As seen in Figure 1~Figure 2, the three omnidirectional wheels are symmetrically assigned on the upper half part of the inner platform, and they have the same angle each there. Additionally, in order to prevent the three wheels from leaving the surface of the sphere shell, we used an elastic support for the omnidirectional wheels. Moreover, for ensuring the stability of our system, we exploit four free-running ball wheels on the bottom of the shell to bear the platform. Similarly, the ball wheels are also mounted on the platform via an elastic support. As for the actuators, three thin motors are used to drive the three omnidirectional wheels to rotate relative to the inner sphere support platform.
Working Principle

The working principle of our robot is as follows:

At the beginning, the three motors drive the omnidirectional wheels to run regulated by a given law. Simultaneously, the running wheels would synthesize a couple to propel the shell to roll via the contacting friction. And in other word, if we control the rotational velocity and direction of the wheels, we could drive the sphere shell in all direction.

After the omnidirectional wheels being driven, because the three wheels are all fixed on the inner platform, there should be a reaction couple acting on the platform by the omnidirectional wheels. With the help of the four free-running ball wheels, the reaction couple would drive the platform to rotate about the geometric center of the shell.

Moreover, if we configure the COG of the platform in different position, the support platform could work either as a rotor of high speed rotating or a pendulum of low speed swinging.

Dynamics

For facilitating our analysis, we assume that our robot had the following characteristics:

\textit{Assumption 1:} the omnidirectional wheels, the shell and the support platform are all rigid body.

\textit{Assumption 2:} the joints between the platform and the omnidirectional wheels are frictionless.

\textit{Assumption 3:} the sphere shell rolls on a horizontal plane without slipping; the omnidirectional wheels run on the inner surface of the shell without slipping.

Additionally, we represent the support platform as $B_1$, the shell as $B_2$, and the omnidirectional wheels as $B_i (i = 3, 4, 5)$. The coordinates of the robot are set up as follow:

- $O - e_1 e_2 e_3 (0)$ is the global coordinate system fixed on the ground;
- $O_1 - e_1^{(1)} e_2^{(1)} e_3^{(1)} (1)$ is the coordinate system of the inner supporting platform $B_1$ and its origin is the geometric center of the sphere shell $B_2$;
- $O_2 - e_1^{(2)} e_2^{(2)} e_3^{(2)} (2)$ is the coordinate system of the shell $B_2$ and its origin is the same as $1$;
- $O_i - e_1^{(i)} e_2^{(i)} e_3^{(i)} (i) (i = 3, 4, 5)$ is the coordinate system of the omnidirectional wheel $B_i$ and the origin locates at its geometric center.

Kinetic Model

We represent $e_i^{(j)} (i = 1, 2, 3, j = 1, 2, 3, \cdots)$ as the $i$th base vector of coordinate $j$, $s_j = \sin(q_j)$, $c_j = \cos(q_j) (i = 1, 2, \cdots)$, and $\dot{q}_i (i = 1, 2, 3)$ as the $i$th Euler angular rate of $B_i$. Because the sphere shell rolls on a horizontal plane, the angular velocity of $B_1$ can be given as:

$$\omega_{B_1}^{(1)} = (c_3 \dot{q}_2 - c_2 s_3 \dot{q}_1) e_1^{(1)} + (s_2 \dot{q}_1 + c_3 \dot{q}_3) e_2^{(1)} + (c_2 c_3 \dot{q}_1 + s_3 \dot{q}_2) e_3^{(1)}. \quad (1)$$

Similarly, the angular velocity of $B_2$ can be given as:

$$\omega_{B_2}^{(2)} = \dot{q}_3 e_1^{(2)} + c_2 \dot{q}_0 e_2^{(2)} + (\dot{q}_4 + s_5 \dot{q}_6) e_3^{(2)}, \quad (2)$$

where $(i = 4, 5, 6)$ is the $(i-3)th$ angular rate of the sphere shell.

Considering $B_i (i = 3, 4, 5)$ rotate about $B_1$, we can get the angular velocity of $B_i (i = 3, 4, 5)$ as:

$$\omega_{B_i}^{(j)} = \hat{R}_i \omega_{B_1}^{(j)} + \dot{q}_l e_3^{(j)} (j = 3, 4, 5), \quad (3) \sim (5)$$

where $\hat{R}_i (i, j = 1, 2, \cdots)$ denotes the rotation transform matric from $\{i\}$ to $\{j\}$, and $\dot{q}_l (i = 7, 8, 9)$ denotes the angular rate of $B_i (i = 3, 4, 5)$, respectively.

According to assumption 3, the velocity of the shell-ground contacting point $P$ is satisfied:
where $\mathbf{R}^{(0)}$ is the position vector of the shell-ground contacting point $P$ in $[0]$. If we assume $\mathbf{v}_{c2}^{(0)} \triangleq \dot{x}_{c2}^{(0)} + y_{c2}^{(0)}$ (and of $z_{c2}$) denote the longitudinal and the lateral velocity of the geometric center of $B_2$, respectively) and consider the first two items in (6), we would get:

$$\dot{q}_5 = (s_5 \dot{x} - c_4 \dot{y})/l, \quad \dot{q}_6 = (c_4 \dot{x} + s_4 \dot{y})/(l + r)c_5,$$

(7)-(8)

where $l$ is the distance between the geometric center of $B_2$ and that of $B_i$ $(i = 3, 4, 5)$; $r$ is the radius of the omnidirectional wheel.

Similarly, we investigate the velocities of the contact point between $B_2$ and $B_i (i = 3, 4, 5)$, then get

$$\dot{q}_i = g_i (6i-5) \dot{x} + g_j (6i-4) \dot{y} + g_l (6i-3) \dot{z} + \sum_{j=5}^{3} g_l (6i-5+j) \dot{q}_j, \quad (i = 1, 2, 3),$$

(9)-(11)

in (9)-(11), $g_j (j = 1, 2, 3, \ldots, 18)$ is a function relating to $q_i (i = 1, 2, 3, 4, 5)$.

Due to the velocity of the geometric center of $B_2$ satisfy $\mathbf{v}_{c2}^{(0)} = \mathbf{v}_{c2}^{(0)}$, under the principle of the relative motion we will get the velocity of $B_i (i, j = 1, 2, \ldots, 5)$ as:

$$\mathbf{v}_{c1}^{(i)} = \mathbf{R}_v \cdot \mathbf{v}_{c2}^{(0)} + \mathbf{w}_{c1}^{(i)} \times \mathbf{I}_{c1}^{(i)}, \quad \mathbf{v}_{c2}^{(i)} = \mathbf{R}_v \cdot \mathbf{v}_{c2}^{(0)} + \mathbf{w}_{c2}^{(i)} \times \left[ \left( \mathbf{I}_{c1}^{(i)} + \mathbf{R}_v \mathbf{J}_{c1}^{(k)} \right) \right] \quad (k = 3, 4, 5),$$

(12)-(15)

where $\mathbf{I}_{c1}^{(i)} (i = 1, 3, 4, 5)$ denotes the position vector in $[i]$ from the center of $B_2$ to that of $B_i$; $\mathbf{I}_{c2}^{(i)} (i = 3, 4, 5)$ is the position vector in $[1]$ from the geometric center of $B_2$ to that of $B_i (i = 3, 4, 5)$.

According to $\mathbf{w}_{c1}^{(i)} (i, j = 1, 2, \ldots, 5)$ in (1)-(5) and $\mathbf{v}_{c1}^{(i)} (i = 1, 2, \ldots, 5; k = 0, 1)$ in (12)-(15), we can calculate the system’s kinetic energy as:

$$T = \sum_{i=1}^{5} \left( \left( \mathbf{w}_{c1}^{(i)} \right)^T \mathbf{J}_{c1} \left( \mathbf{w}_{c1}^{(i)} \right) + \left( \mathbf{v}_{c1}^{(i)} \right)^T \mathbf{M}_{c1} \left( \mathbf{v}_{c1}^{(i)} \right) \right)/2,$$

(16)

where $\mathbf{J}_{c1} \left( \mathbf{M}_{c1} \right)$ $(i = 1, 2, \ldots, 5)$ is the inertial matrix (mass matrix) of $B_i (i = 1, 2, \ldots, 5)$, respectively. By substituting (7)-(11) into $T$, we will get another form of the kinetic energy $\tilde{T}$.

**Dynamic Model**

The external force acting on our spherical robot consists of the driving torque of $B_i (i = 3, 4, 5)$ and the gravity. We consider the COM of $B_i (i = 1, 3, 4, 5)$, which can be represented as:

$$\mathbf{l}_{c1}^{(i)} = \left( m_1 \mathbf{l}_{c1}^{(i)} + m_2 \mathbf{l}_{c2}^{(i)} + \mathbf{R}_v \mathbf{l}_{c3}^{(i)} + m_3 \mathbf{l}_{c4}^{(i)} + m_4 \mathbf{l}_{c4}^{(i)} + m_5 \mathbf{l}_{c5}^{(i)} \right)/m_i,$$

(17)

where $m_i = m + m_1 + m_2 + m_3 + m_4 + m_5$. If we defined $\mathbf{h} \triangleq \mathbf{R}_v \mathbf{l}_{c1}^{(i)}$ $[3]$ $((*)[3]$ is the 3rd item of the vector $*)$, we will find $\mathbf{h}$ is the function of $q_i (i = 2, 3)$. And if we take the first time derivative of $\mathbf{h}$, we will get:

$$\dot{\mathbf{h}} = f_1 \mathbf{q}_2 + f_2 \mathbf{q}_3,$$

(18)

in (18), $f_j (i = 1, 2)$ denotes the explicit function of $q_i (i = 2, 3)$ and $m_j (j = 1, 3, 4, 5)$. With (18), we define $\mathbf{r}_g \triangleq \left( \mathbf{r}_{g2} \mathbf{r}_{g3} \right)^T$, in which $\mathbf{r}_{gk} = m_l g \cdot f_{(k-1)} (k = 2, 3)$.

Moreover, according to (9)-(11), we define a transform matrix: $\mathbf{R}_c \triangleq \left( g_{ij} \right)_{3 \times 6}$. Consequently, the generalized forces of the system can be calculated as
where \( \mathbf{Q} = (0 \ 0 \ 0 \ \tau_7 \ \tau_8 \ \tau_9)^T \), \( \tau_i \) \((i=7,8,9)\) is the driving torque of \( B_j \) \((j=3,4,5)\). Considering the following form of Chaplygin equation

\[
\frac{d}{dt} \mathbf{\tilde{T}} - \sum_{\alpha=1}^{r} \sum_{\beta=1}^{s} \frac{\partial B_{\alpha,\beta,\sigma}}{\partial \mathbf{q}_{\sigma}} - \sum_{\alpha=1}^{r} \frac{\partial B_{\alpha,\beta,\nu}}{\partial \mathbf{q}_{\nu}} \mathbf{v} = \mathbf{\tilde{Q}},
\]

(20)

where \( T \) is the kinetic energy and \( \mathbf{\tilde{T}} \) is the kinetic energy by substituting noholonomic constrains into \( T \); \( B_{\alpha,\beta,\sigma} \) is the \( \sigma \)th coefficient of the \( \beta \)th noholonomic constrain; \( \mathbf{q}_{\nu} \) and \( \mathbf{q}_{\sigma} \) are the generalized coordinates of the system; \( \mathbf{\tilde{Q}} \) is the \( \sigma \)th generalized force of the system.

We can get the system’s dynamics:

\[
\mathbf{D}(\mathbf{q})\mathbf{\dot{q}}_{\sigma} + \mathbf{C}(\mathbf{q}, \mathbf{\dot{q}}_{\sigma})\mathbf{\ddot{q}}_{\sigma} + \mathbf{G}(\mathbf{q}) = \mathbf{Q},
\]

(21)

in (21), \( \mathbf{D}(\mathbf{q}) \in \mathbb{R}^{66} \), \( \mathbf{C}(\mathbf{q}, \mathbf{\dot{q}}_{\sigma}) \in \mathbb{R}^{66} \), and \( \mathbf{G}(\mathbf{q}) \in \mathbb{R}^{64} \) denote the inertia, centripetal-Coriolis, and gravity terms; \( \mathbf{q}_{\sigma} \) and \( \mathbf{q} \) are two kinds of generalized coordinates, which are represented as follow:

\[
\mathbf{q}_{\sigma} = (x \ y \ q_1 \ q_2 \ q_3 \ q_4 \ q_5)^T \quad \text{and} \quad \mathbf{q} = (q_1 \ q_2 \ q_3 \ q_4 \ q_5)^T.
\]

Equation (21) indicates our robot is an under-actuated system with six independent velocities, and the longitudinal displacement and lateral displacement \((x, y)\) of the geometric center of \( B_1 \), the yaw angle \((q_3)\) of \( B_1 \) are under-actuated; there are three driving inputs in \( q_i \) \((i=7,8,9)\), so we could regulate control-force \( \tau_i \) \((i=7,8,9)\) to control the trajectory \((x, y, q_4)\) of the sphere shell.

### Positioning Controller

Our control objective is to regulate the position of the COM of the sphere shell to a constant desired setpoint: \((x \ y \ q_4)^T = (x_d \ y_d \ 0)^T\). Accordingly, we should define an error signal as:

\[
(\epsilon_1 \ \epsilon_2 \ \epsilon_3 \ \epsilon_4 \ \epsilon_5) = (x-x_d \ \dot{x} \ y-y_d \ \dot{y} \ q_4 \ \dot{q}_4)^T.
\]

(22)

We investigate the first three items of the dynamical model (Seeing Eq.(21)):

\[
\mathbf{D}_{a1}(\mathbf{q})\mathbf{\dot{\tilde{q}}}_{\tilde{u}} + \mathbf{D}_{a1}(\mathbf{q})\mathbf{\ddot{q}}_{\tilde{u}} + \mathbf{F}_i(\mathbf{q}, \mathbf{\dot{q}}_{\sigma}) = 0,
\]

(23)

where \( \mathbf{\dot{\tilde{q}}}_{\tilde{u}} = (\ddot{x} \ \ddot{y} \ \dot{q}_4)^T \), \( \mathbf{\ddot{q}}_{\tilde{u}} = (\ddot{q}_7 \ \ddot{q}_8 \ \dot{q}_6)^T \), and \( \mathbf{F}_i(\mathbf{q}, \mathbf{\dot{q}}_{\sigma}) \in \mathbb{R}^{3 \times 1} \) is the first three items of \( \mathbf{C}(\mathbf{q}, \mathbf{\dot{q}}_{\sigma})\mathbf{\dot{q}}_{\sigma} + \mathbf{G}(\mathbf{q}) \), \( \mathbf{D}_{a1}(\mathbf{q}) \), \( \mathbf{D}_{a2}(\mathbf{q}) \) \( \mathbf{D}_{a3}(\mathbf{q}) \) \( \in \mathbb{R}^{3 \times 3} \) are a sub-matric of \( \mathbf{D}(\mathbf{q}) \). Then we will get

\[
\mathbf{\dot{\tilde{q}}}_{\tilde{u}} = -(\mathbf{D}_{a1}(\mathbf{q}))^{-1}(\mathbf{F}_1(\mathbf{q}, \mathbf{\dot{q}}_{\sigma}) \mathbf{D}_{a1}(\mathbf{q})\mathbf{\dot{q}}_{\tilde{u}}).
\]

(24)

Substituting (24) into the last three items of the dynamical model, we will get:

\[
\left( \mathbf{D}_{a2}(\mathbf{q}) - \mathbf{D}_{a2}(\mathbf{q})\left(\mathbf{D}_{a1}(\mathbf{q})^{-1}\mathbf{D}_{a1}(\mathbf{q})\right)\right)\mathbf{\dot{\tilde{q}}}_{\tilde{u}} - \mathbf{D}_{a2}(\mathbf{q})\left(\mathbf{D}_{a1}(\mathbf{q})^{-1}\mathbf{F}_1(\mathbf{q}, \mathbf{\dot{q}}_{\sigma})\right) + \mathbf{F}_2(\mathbf{q}, \mathbf{\dot{q}}_{\sigma}) = \mathbf{\tau}_a,
\]

(25)

where \( \mathbf{F}_2(\mathbf{q}, \mathbf{\dot{q}}_{\sigma}) \in \mathbb{R}^{3 \times 1} \) is the last three items of \( \mathbf{C}(\mathbf{q}, \mathbf{\dot{q}}_{\sigma})\mathbf{\dot{q}}_{\sigma} + \mathbf{G}(\mathbf{q}) \), and \( \mathbf{D}_{a2}(\mathbf{q}) \), \( \mathbf{D}_{a3}(\mathbf{q}) \) \( \in \mathbb{R}^{3 \times 3} \) are the corresponding sub-matric of \( \mathbf{D}(\mathbf{q}) \). If we define \( \mathbf{\dot{\tilde{q}}}_{\tilde{u}} \equiv \mathbf{v} \), we will get the input torque as:

\[
\mathbf{\tau}_a = \left( \mathbf{D}_{a2}(\mathbf{q}) - \mathbf{D}_{a2}(\mathbf{q})\left(\mathbf{D}_{a1}(\mathbf{q})^{-1}\mathbf{D}_{a1}(\mathbf{q})\right)\mathbf{v} - \mathbf{D}_{a2}(\mathbf{q})\left(\mathbf{D}_{a1}(\mathbf{q})^{-1}\mathbf{F}_1(\mathbf{q}, \mathbf{\dot{q}}_{\sigma})\right) + \mathbf{F}_2(\mathbf{q}, \mathbf{\dot{q}}_{\sigma}) \right),
\]

(26)
where the virtual control law are defined as: \( v = -k_pe_i - k_p\dot{e}_i, e_i = (e_1, e_3, e_5)^T, \quad \dot{e}_i = (e_2, e_4, e_6)^T, \quad k_p, k_d \in \mathbb{R}^{3 \times 3}, \) and \( k_p = \text{diag}(k_p1, k_p2, k_p3), \quad k_d = \text{diag}(k_d1, k_d2, k_d3), \quad k_{pi}, \quad k_{di}(i=1,2,3) \) are positive constant control gains. With (26), we will obtain the following theorem:

**Theorem:** The controller (26) ensures the system: \( \lim_{t \to 0} (x(t) \quad \dot{x}(t) \quad y(t) \quad \dot{y}(t) \quad q_d(t) \quad \dot{q}_d(t))^T = (x_d \quad y_d \quad 0 \quad 0 \quad 0 \quad 0)^T, \) where \( (x_d \quad y_d)^T \) is the desired position of the COM of the sphere shell.

**Proof:** In order to prove the Theorem, firstly, we define:

\[
\dot{e}_i \triangleq \dot{q}_{da} - \dot{q}_{rd} = \ddot{q}_{da} = v,
\]

where \( \dot{q}_{rd} = (\ddot{x} \quad \ddot{y} \quad \ddot{q}_d)^T = (0 \quad 0 \quad 0)^T. \) According to (27), we will obtain:

\[
\ddot{e}_i + k_p \dot{e}_i + k_r e_1 = 0,
\]

In (28), if we set suitable \( k_d, k_p, \) we will get: \( \lim_{t \to 0} \dot{e}_i(t) = 0, \lim_{t \to 0} \dot{e}_d(t) = 0, \lim_{t \to 0} e_1(t) = 0. \) That is

\[
\lim_{t \to 0} \dot{q}_{da}(t) = 0, \lim_{t \to 0} q_{rd}(t) = (x_d \quad y_d \quad 0)^T.
\]

After substituting (29) into (23), we can obtain the zero dynamics of the robot system:

\[
D_{cl}(q)\ddot{q}_{da} + F_1(q, \dot{q}_d) = 0.
\]

Considering (30) around balanced point (\( q_1 = 0, q_2 = \arcsin(\sqrt{3}/3), q_3 = -\arcsin(\sqrt{2}/2) \)), we will obtain: \( F_1(q, \dot{q}_d) = 0, \quad F_2(q, \dot{q}_d) = 0; \quad g_4 = g_6 = 0, \quad g_5 = \lambda \) (in which \( \lambda \) is a constant), \( g_{10} = -g_{12}, \quad g_{11} = 0, \quad g_{16} = g_{18} = -g_{17}/2. \) As a result, we can utilize (30) to show that \( \dot{q}_{da} = 0, \) and utilize (26) to show \( \tau_a = 0. \) Eventually, we can get that \( \dot{q}_{rd} = \gamma \) (where \( \gamma \in \mathbb{R}^{3 \times 1} \) is a constant vector).

With (7)–(11), it is clear that \( \dot{q}_1 = c \cdot \dot{q}_2 = \dot{q}_3 = \dot{q}_5 = \dot{q}_6 = 0 \) (where \( c \) is a constant). So we can get:

\[
\lim_{t \to 0} \left( \dot{q}_1(t) \dot{q}_2(t) \dot{q}_3(t) \dot{q}_4(t) \dot{q}_5(t) \dot{q}_6(t) \right)^T = (c \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)^T.
\]

Summarily, with (29) and (31), we can conclude that is the end of our proof.

### Control Simulation

We preset the desired objective as: \( (x_d \quad y_d)^T = (1-e^{-0.01t} \quad 0)^T \quad (m), \quad q_{rd} = 0(rad) \). Considering the controller (26), we tuned the virtual control gains until there a better performance was achieved: \( k_{p1} = 0.022, k_{p2} = 0.015, \quad k_{p3} = 0.010, \quad k_{d1} = 0.025, \quad k_{d2} = 0.020, \quad k_{d3} = 0.010. \)

The structural parameters of our system were set as: \( r = 0.06(m); \quad l = 0.25(m); \quad L_1 = 0.20(m) \) (the distance between the shell’s center and the COG of the platform); \( L_2 = 0.25(m) \) (the distance between omnidirectional wheel’s center and COG); \( m_1 = 17.57(kg) \) (\( m_2 = 2.87(kg), \quad m_3 = 0.45(kg) \)); \( J_{B1} = \text{diag}(0.780, 0.780, 0.780) \quad (kg \cdot m^2), J_{B2} = \text{diag}(0.130, 0.130, 0.130) \quad (kg \cdot m^2), \quad J_{B3} = \text{diag}(0.001, 0.001, 0.002) \quad (kg \cdot m^2) \) (the inertia matrix of the platform(shell, omnidirectional wheel)).

In our experiment, the initial conditions were set to zero. The results are shown in Figure 3~Figure 7.
Based on the results illustrated in Figs. 3~5, it is clear that the control laws describing in (26) exhibit certain performance to approach the desired objective. The sphere shell stands by at the desired setpoint but the platform rotates around the vertical axis eventually. One reason for this result is that the variables of the shell (x and y) are feedback to the controller but the variables of the three driving omnidirectional wheels (qi, i = 7, 8, 9) are not to it. Accordingly, the three omnidirectional wheels’ rotation converging to an equal speed value. Moreover, the maximum driving torque of the three driving omnidirectional wheels is less than 80N.m, and the input angular velocity of which is less than 100rad/s.

Conclusions

The first work of this paper is that we proposed a new type of spherical robot which is driven by three omnidirectional wheels. The second work of this paper is that we developed a dynamical model by using Chaplygin equation and investigated the positioning problem by using partial linearization controller for the system. Our modelling results show that the spherical robot is an under-actuated system of six independent velocities and three control-torque inputs. With numerical simulation, we validated that the feasibility of our model and the reliability of our controller.

Our next work should focus on performing the control experiments on the principle prototype so as to further testify the characteristics of the robot system.

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