A Semi-empirical Modulation Model on Non-cavitation Noise of Underwater Counter-rotation Propellers

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Abstract. Underwater counter-rotation propeller non-cavitation noise has an obvious modulation characteristic which is due to the interaction of flow and blade, the modulation characteristic is critical for underwater target identification. Based on generalized acoustic analogy theory, a semi-empirical modulation model has been presented in this paper to describe its non-cavitation noise characteristics. Theoretical analysis has been verified by numerical simulation and cavitation tunnel experiment results. The numerical simulation has shown the directivity characteristic of typical modulated line-spectra. The experiment results have agreed with theoretical values. The modulation model of counter-rotation propeller is beneficial to the prediction modulation characteristics and identification of underwater high-speed vehicles.

Introduction

The radiated noise characteristics of underwater high-speed vehicle are important research field in underwater acoustic warfare which is a key factor for detection and recognition of underwater high-speed vehicles. For medium and high speed underwater vehicle, the propulsor will be the main noise source. The counter-rotation propeller is a good propulsor which is commonly used to drive the underwater high speed vehicles. Theoretically analysis of counter-rotation propeller radiated noise mechanics is still a challenging work and previous works mainly focus on the aviation field. A frequency domain model and a broad-band spectrum model of its radiation noise are presented by Hanson¹ and Blandeau². Additional improvement to Hanson’s model is presented by Parry with application in engineering prediction and Zhu³ with application in underwater counter-rotation propeller line-spectrum prediction. To our knowledge, no modulation models of underwater counter-rotation propeller radiation noise have been published in the open literature. This paper presents a semi-empirical modulation model for underwater counter-rotation propeller non-cavitation noise. Additional numerical simulation and experiment are presented to verify theoretically analysis. Finally, several conclusions are presented in this paper.

Sound Pressure Spectrum of Non-cavitation Noise on Counter-rotation Propeller

From the generalized acoustic analogy equation [⁴], the radiation noise of underwater propeller can be described by monopole, dipole and quadrupole, as shown in the following equation.

\[
\rho = \frac{1}{\omega} \int \int \frac{\partial^2 G}{\partial y \partial \tau} T_{ij} dS(y) d\tau - \frac{1}{\omega} \int \int \frac{\partial G}{\partial y} dS(y) d\tau + \frac{1}{\omega} \int \int \rho(y) \frac{\partial G}{\partial \tau} dS(y) d\tau
\]

(1)

Where \( v_n \) and \( f_i \) take the normal direction as positive of the moving object outer surface, \( c_0 \) is the sound speed of water, \( G \) denotes a spatial Green function, \( T_{ij} \) is the lighthill stress tensor, \( \rho \) is the medium density perturbation, \( \tau \) is the time variable for the sound source, \( S(\tau) \) is the integration area.
of sound source, \( \mu(\tau) \) is the volume of the space other than the propeller. We focus on the generation and modulation mechanism of underwater counter-rotation propeller non-cavitation noise. The unsteady force acting on the blade is the main noise source. Through the analysis of the characteristics of the flow field, it can be obtained that the non-cavitation radiation noise of underwater counter-rotation propeller mainly generated by the following seven kinds of effects: Interaction between non-uniform inflow and front propeller leading edge, the effect of large scale vortices near the blade, the effect of secondary flow on the rear blade, the effect of unsteady pressure on the front blade which is formed by the post-pulp suction of rear blade, the interaction between eddy vortex of the front blade and leading edge of rear blade, the interaction between trailing edge shedding vortex of the front blade and leading edge of rear blade and the interaction between hub shedding vortex of the front blade and the rear blade. These seven kinds of effects can be easily divided into two categories which are interference effect and circumferential harmonics flow effect. As shown in Fig. 1.

![Figure 1. Schematic of interference effect (left) and circumferential harmonics flow effect (right).](image)

The far-field sound pressure spectrum of rear propeller which caused by the interference of the front propeller can be written as follow, the detailed derivation refer to [5].

\[
P_{21}(r, \theta) = \frac{\mu_2 c_2^3 B_2 \sin \theta_2}{8 \pi c_2 r_2} \sum_{i} \sum_{j} \exp \left\{ i \left[ (m B_2 - k_2^i B_2) \phi_2 - \frac{\pi}{2} + k_2^i B_2 \Omega + m B_2 \Omega \frac{r_2}{1 - Ma_2 \cos \theta_2} \left( \frac{r_2}{c_2} - 1 \right) \right] \right\} \int_{-\infty}^{\infty} M_{21}^2 \cdot \exp[i(\phi_2^{(2)} + \phi_2^{(2)})] J_{\omega_2} \cdot \Omega_2 \cdot \left( \frac{k_2^i B_2 M_{21} \omega_2 + m B_2 M_{21} \omega_2}{(1 - Ma_2 \cos \theta_2)} \right) \cdot [k_2^{(2)} \psi_{\alpha_2}(k_2^{(2)}) + k_2^{(2)} \psi_{\alpha_2}(k_2^{(2)})] \cdot \left. \cdots \right|_{-\infty}^{\infty} \tag{2}
\]

Where subscript 2 represent the rear propellers. \( B_2 \) denote the blade number of rear propeller. \( D_2 \) represents the diameter of the rear propeller. \( r_2 \) represents the distance between source point and the far-field observation point. \( b_2 \) represents the chord length. \( MCA \) and \( FA \) are the skewing and pitching distance. \( \phi_2^{(2)} \) represents the calculated initial phase angle. \( Ma_2^{(2)}, Ma_2^{(2)}, \) and \( Ma_{tip} \) represent the advancing Mach number, the blade section Mach number and the blade tip Mach number. \( n, m, \) \( k_2^i \) and \( k_2^{(2)} \) represent the harmonic order. \( C_{Dk}, C_{Lk}, C_{Dw}, \) and \( C_{Lw} \) represent the lift and resistance coefficients. \( \Omega \) represents the blade section shape function. Take the influence of circumferential harmonic flow field into consideration. The far field sound pressure spectrum of the rear propeller due to the action of the harmonic flow field can be written in the same way. The far field sound pressure spectrum of underwater counter-rotation propeller can be obtained by superimposing the far field sound pressure spectrum of the front and rear propellers.

### Semi-empirical Modulation Model of Counter-rotation Propeller in Non-cavitation Condition

Radiated noise modulation characteristic has always been very beneficial to describe underwater target. The semi-empirical localized time-varying power spectrum for underwater counter-rotation propeller can be described as follow.
\[ G(t, f) = K \cdot m(t) \cdot m(f) \cdot G_e(f) \]  

(3)

Where \( K \) is the proportional coefficient, \( m(t) \) is the modulation function which is only related to time term, \( m(f) \) is the modulation depth which is a function of the line-spectrum frequency. \( G_e(f) \) represents the power spectrum of the Ergodic Gaussian process. Only consider the modulation characteristics of the rear propeller which caused by the interference of the front propeller. The proportional coefficient, modulation function and modulation depth function can be written as follow.

\[ K = \frac{\rho \epsilon^2 B_i D_i}{8\pi} \]

(4)

\[ m(t) = \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} e^{i(mB_z-kB_1)\phi} e^{-i\frac{kB_2B_3}{1-M_\alpha \cos \theta}} \]

(5)

\[ m(k^f_1, m^f_2) = \int_{\text{root}}^{\text{root}} J_{mB_2-kB_1}(2\pi) \frac{k^f_1 B_1 f_1 + mB_2 f_2}{c_e (1-M_\alpha \cos \theta)} e^{-i\frac{kB_2B_3 + mB_2B_3}{1-M_\alpha \cos \theta}} \]

(6)

Based on equation (5), a prediction can be made that the modulation frequency is \((mB_z - kB_1) / 2\pi \) when the eigenvalue is \((mB_z - kB_1)\) in \( \phi \) direction. \( 1/(1-M_\alpha \cos \theta) \) represents the Doppler frequency shift. The modulation line-spectrum can be written as \( f_{\text{mod}} = m^f_1 + n^f_2 \), \( m, n = 0, \pm 1, \pm 2, \ldots \). Equation (6) represents the modulation depth of line-spectrum in the radiated noise. Equation (6) also shows that the different modulated line-spectrum at the same observation point has different directivity. From above analysis, an semi-empirical modulation model of CRP has been built based on equation(3)–(6).

**Numerical Simulation and Experiment Results**

As shown in Fig. 2, the directivity characteristic of typical modulated line-spectra are calculated when the rotation speed is 14rps. It can be seen that the modulated line-spectrum \( f_1 + f_2 \) has obvious dipole characteristic; the modulated line-spectrum \( 3f_1 + 2f_2 \) has obvious quadrupole characteristic. The higher the harmonic order is, the sharper the directivity becomes.

![Figure 2. The directivity characteristics of typical modulated line-spectrum.](image-url)

The non-cavitation radiated noise experiment of counter-rotation propeller model is carried on the large-scale high-speed cavitation tunnel in China Ship Science Research Center. The basic parameters
of counter-rotation propeller are shown in Table 1. The installation diagram of model is shown in Fig. 3.

Table 1. The basic parameters of experimental model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Front propeller</th>
<th>Rear propeller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter /mm</td>
<td>305.2</td>
<td>299.3</td>
</tr>
<tr>
<td>Num. of blades</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>(P/D_{b_{78}})</td>
<td>1.291</td>
<td>1.287</td>
</tr>
<tr>
<td>Disk surface ratio</td>
<td>0.303</td>
<td>0.379</td>
</tr>
<tr>
<td>Direction of rotation</td>
<td>left</td>
<td>right</td>
</tr>
<tr>
<td>Profile type</td>
<td>NACA 66</td>
<td>NACA 66</td>
</tr>
</tbody>
</table>

The experiment data is processed in 3 kHz~40 kHz by square demodulation principle. The results are shown in Figs. 4.

Figure 3. Schematic diagram of install propeller model.

Figure 4. The modulation in whole band spectrum in 7rps.

Table 2. Comparison of theoretical and experimental values for modulation depths of typical modulated line-spectrum of CRP in 7r/s.

<table>
<thead>
<tr>
<th></th>
<th>(f_1)</th>
<th>(f_2)</th>
<th>(3f_1+2f_2)</th>
<th>(9f_1+2f_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical value</td>
<td>0.53</td>
<td>0.02</td>
<td>0.72</td>
<td>0.49</td>
</tr>
<tr>
<td>Experimental value</td>
<td>0.66</td>
<td>0.015</td>
<td>0.61</td>
<td>0.57</td>
</tr>
<tr>
<td>Error(%)</td>
<td>24.5%</td>
<td>25%</td>
<td>15.3%</td>
<td>16.2%</td>
</tr>
</tbody>
</table>

It can be seen from the dotted lines in these figures that none modulation effect exist in the radiation noise with the absence of counter-rotation propeller, however, blade passing frequency, harmonics of blade passing frequency, shaft frequency and combination of them are the mainly modulated line-spectrum of counter-rotation propeller radiated noise. It is found that the difference between the
theoretical values and the experiment values of typical modulated line-spectrum are less than 25% which means that the semi-empirical modulation model can predicate the modulation depth of typical modulated line-spectrum.

Summary
In this paper, a semi-empirical modulation model of underwater counter-rotation propeller is deduced by the generalized acoustic analogy method. Then the modulation mechanism is presented and verified by experiment. The results of experiment and numerical simulation have verified the semi-empirical modulation model. The semi-empirical modulation mechanism is beneficial for location, recognition and radiation noise simulation of underwater high-speed vehicles.

Acknowledgement
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References