The Global Kondaurov Double Porosity Model

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Abstract. In this paper we derive a non-equilibrium matrix imbibition equation for the homogenized Kondaurov double porosity model which is valid for arbitrary relaxation times. This equation involves the real saturation and Kondaurov's non-equilibrium parameter, i.e., the principal unknowns of the model. The numerical simulation of the obtained result is also presented.

Introduction

The paper is devoted to the study of the Kondaurov double porosity model which modelizes a non-equilibrium two-phase flow of immiscible incompressible fluids in fractured-porous media. This model was obtained recently in [10] and then revisited in the paper [11] in order to simplify the local problem formulated in terms of the phase pressures and the real saturation reducing it, for small relaxation times, to the so called non-equilibrium imbibition equation.

The mathematical modeling and numerical simulation of two-phase flow in fractured porous reservoirs is important for many practical problems such as petroleum reservoir engineering, soil science etc. We observe that the physical and mathematical models presented in the existing literature are, in fact, the equilibrium ones. However, the experimental studies [4] and the engineering applications of the two-phase flow models [13] show that these models are not sufficient to describe the two-phase flow process in porous media in a proper manner. For example, as it is underlined in [4], the reason for a slow oil recovery can be a non-equilibrium behavior of the capillary pressure. This shows the importance of the involving of the non-equilibrium phenomena in the mathematical modeling of the two-phase flow in porous media. Here we also refer to [7] for the physical description and explanation of the non-equilibrium phenomena in porous media.

The homogenization and numerical simulation of single- and two-phase flow in double porosity media modeling real fractured porous reservoirs is an important problem attracting the researches during many years. There exists an extensive literature describing the analytical methods applied to this kind of problem and the results of the numerical simulation as well. We will not attempt a literature review here but will merely mention a few references on the subject. We refer, e.g., to the monographs [8, 12], and the references therein. It is important to notice that the mesoscopic models of single- and two-phase flow in double porosity media considered in these works are locally equilibrium ones. However, the homogenized models obtained exhibit the non-equilibrium behavior due to the additional "source terms" describing the memory effect of the corresponding model (see [2, 5, 12, 15]). Moreover, as it follows from [1, 6], it is possible to show that the capillary pressure of the global model is, in fact, a non-equilibrium one. However, we observe that only in a few papers the authors start with a locally non-equilibrium model. This is done in [2,13], where the homogenization of Barenblatts and Hassanizadehs flow models (see [3]) and in [10, 11], where the homogenization of Kondaurov's model is considered. Notice that the Barenblatt, Hassanizadeh, and Kondaurov flow models are discussed in details in the papers [9, 10].

In this paper we focus our attention on the Kondaurov homogenized non-equilibrium double porosity type model obtained in [10]. More precisely, we are interesting in the representation of the local problem in a form which will be more appropriate for the numerical simulation. The local problem in [10] is formulated in terms of the phase pressures and a real saturation. Knowing that the numerical methods are very sensitive to the choice of the governing equations form we are going to simplify this problem reducing it to a non-equilibrium imbibition equation.
In this section we formulate the mesoscopic flow equations of the Kondaurov model and then introduce the corresponding homogenized model obtained earlier in [10]. We consider a reservoir $\Omega \subset \mathbb{R}^d$ with a periodic structure (see Fig. 1). More precisely, we scale this structure by a parameter $\varepsilon$ representing the ratio of the cell size to the whole $\Omega$ and assume that $\varepsilon \downarrow 0$. Let $Y \equiv (0, 1)^d$ be a basic cell of a fractured porous medium. We assume that $Y$ is made up of two homogeneous porous media $Y_m$ and $Y_f$ corresponding to the parties of the mesoscopic domain occupied by the matrix block and the fracture, respectively. Thus $Y = Y_m \cup Y_f \cup \Gamma_{fm}$, where $\Gamma_{fm}$ is the interface between two media. Let $\Omega^\varepsilon$ denotes the open set corresponding to the porous medium with index $\ell = "m"$ or "f". Then $Y = Y^{\varepsilon}_m \cup Y^{\varepsilon}_f \cup \Gamma^{\varepsilon}_{fm}$, where $\Gamma^{\varepsilon}_{fm} = \partial \Omega^{\varepsilon}_f \cap \partial \Omega^{\varepsilon}_m \cap \Omega$.

Fracture and matrix porosities $0 < \Phi_f, \Phi_m < 1$ are constants and do not depend on $\varepsilon$; fracture and matrix permeabilities $0 < K_f, K_m < +\infty$ are constants and do not depend on $\varepsilon$.

Let $S, P_w, P_n$ denote the homogenized wetting liquid saturation, the wetting and nonwetting liquid pressures, respectively. Let $\Phi^*$ be effective porosity, $\Phi^* \equiv \Phi |Y_f|/|Y_m|$, $K^*$ is a standard homogenized tensor (see [10, 11, 14]).

The homogenized system in $\Omega^\varepsilon$ has the form:

$$\begin{cases}
\Phi^* \frac{\partial S}{\partial t} - \text{div}\{ K^* \lambda_{fw}(S) \nabla P_w \} = - \frac{\Phi_m}{|Y_m|} \int_{Y_m} \frac{\partial S}{\partial t} (x, y, t) \, dy;

- \Phi^* \frac{\partial S}{\partial t} - \text{div}\{ K^* \lambda_{fn}(1-S) \nabla P_n \} = \frac{\Phi_m}{|Y_m|} \int_{Y_m} \frac{\partial S}{\partial t} (x, y, t) \, dy;

P_c(S) = P_n - P_w \quad \text{with} \quad \Phi_f P_c(S) \equiv (\gamma_f + M_f - \alpha_f^2/\beta_f) - (\gamma_f - \alpha_f^2/\beta_f) S.
\end{cases}$$

The equations for flow in the matrix block $Y_m$ are given by:

$$\begin{cases}
\Phi_m \frac{\partial S}{\partial t} - \text{div}_y \{ K_m \lambda_{m,w}(S, \xi_m) \nabla_y p_w \} = 0;

- \Phi_m \frac{\partial S}{\partial t} - \text{div}_y \{ K_m \lambda_{m,n}(S, \xi_m) \nabla_y p_n \} = 0;

p_c(S, \xi_m) = p_n - p_w \quad \text{with} \quad \Phi_m p_c(S, \xi_m) \equiv \gamma_m + M_m (1-S) - \alpha_m S.
\end{cases}$$

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Here $s, p_w, p_n$ denote the wetting liquid saturation, the wetting and nonwetting liquid pressures in the matrix block $Y_m$, respectively; $\alpha, \beta, \gamma, M > 0$ are the constitutive parameters and do not depend on $\varepsilon; \lambda_{w}, \lambda_{n}$ denote the wetting and nonwetting liquid mobilities; $\xi$ denotes the local non-equilibrium parameter in $Y_m$ defined by:

$$\frac{\partial \xi}{\partial t} = \frac{1}{\tau_m} \Lambda(s, \xi) \text{ with } \Lambda(s, \xi) = \frac{\alpha_m}{\beta_m} (1 - s) - \xi.$$  

**Remark 1.** Notice that in the case of the equilibrium flow the macroscopic model (1)-(3) is exactly (up to a specified capillary pressure) the well known homogenized double porosity model for the immiscible incompressible two-phase flow in porous media considered by many authors (see, e.g., [6, 15]).

The local problem (2) can be reduced to a new problem, for a non-equilibrium imbibitions equation given in terms of the saturation $s$ and the parameter $\xi$ being a functional of $s$ (see [14]):

$$\left\{ \Phi \frac{\partial s}{\partial t} + K_m \text{div} \left\{ \frac{\lambda_m(s, \xi)}{\lambda_m(s, \xi)} \nabla p_e(s, \xi) \right\} = 0 \text{ in } Y_m \times (0, T); \\
p_e(s, \xi) = p_e(s) \text{ on } \Gamma_m; \\
s(x, y, 0) = s^{\text{init}}(x) \text{ in } Y_m. \right\}$$  

**Remark 2.** Notice that equation (4) is integro-differential. This fact shows explicitly the impact of the capillary non-equilibrium on the mass exchange between the fracture system and blocks via the source terms in (1).

![Figure 2. Saturation distribution for $\tau_m = 0$ (a), $\tau_m = 2$ (b) and $\tau_m = 10$ (c).](image)
Concluding Remarks
In this paper we generalize the results of the paper [11]. Namely, we obtain a general form of the non-equilibrium matrix imbibition equation for the Kondaurov double porosity model given by (4). It is formulated in terms of the real saturation $s$ and Kondaurov's non-equilibrium parameter $\xi_m$, namely, the principal unknowns of the flow model. In contrast to the non-equilibrium imbibition equation from [11], equation (4) is valid for arbitrary relaxation times. The numerical simulation of equation (4) shows that the intensity of the matrix-fracture exchange increases when the relaxation time $\tau_m$ increases. Moreover, the imbibition process becomes faster.

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References


