Low-complexity Channel Estimation for Large-scale Receiving Antenna Systems Based on PASTd

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Abstract. Subspace estimation is an important procedure when we make predictions about the direction of arrival (DOA). Low-complexity projection approximation and subspace tracking of deflation (PASTd) is able to estimate signal subspace without computing eign-value decomposition of the cross correlation matrix or singular value decomposition of the received signal. Furthermore, we regard the channel estimation as the combination of angular information and path attenuation. In this paper, we discuss PASTd comes into play in DOA and channel estimation. Simulation results in MATLAB verify the effectiveness in angular and channel estimation.

Introduction

In massive received antenna array system, not a few of algorithms have been studied deeply to capture targets’ angle information. The methods of two-dimensional DOA estimation includes the 2D-MUSIC algorithm [1], 2D-ESPRIT algorithm [2], propagator method (PM) algorithm [3] and so on. The same point lies in the necessary procedure of solving the signal subspace. However, the way in these algorithms is to calculate covariance matrix and then obtain signal and noise subspace with the eign-decomposition of covariance matrix. In recent years, subspace tracking algorithm such as PAST [4] and PASTd [5] have attracted wide attention because of their low complexity.

Massive antenna arrays have been accepted and will be surely utilized in mmWave communication system. Because of the short wavelength (less than 10 millimeter), the receiver can integrate dozens of hundreds of antennas in a small area [6]. Even though huge antenna arrays bring about a great number of advantages [7,8,9], an nonnegligible problem is the increased complexity resulting from the matrix dimension concerning antenna numbers. Therefore, decreasing algorithm complexity becomes the key of our research in this paper. We exploit PASTd [4,14] combined with the idea of ESPRIT algorithm for DOA based on two-dimension rectangular antenna array. This method need not much computational cost comparing to traditional ESPRIT.

The channel estimation can be regarded as the combination of angular information and path attenuation [10,11]. Angular estimation accuracy has an influence on channel estimation. It is necessary to analyze how to balance the accuracy and complexity. The main contribution of this paper lies in applying PASTd algorithm to DOA estimation and then studying the channel estimation performance based on PASTd. The pilots are utilized to estimate channel state information [12] and we compare our algorithm with traditional Least Square (LS) channel estimation algorithm under the condition of additive pilot sequence.

The rest of this paper is organized as follows. In section II we introduce the data model and the directional matrix of rectangular antenna array. Moreover, a low-complexity algorithm on angles and
channel estimation is analyzed in section III. Simulation results based on MATLAB are presented in section IV. Finally, we have an inclusion in section V.

Notation: \((\cdot)^T\), \((\cdot)^H\), \((\cdot)^{-1}\), \((\cdot)^{\dagger}\) and \((\cdot)^*\) denote transpose, conjugate-transpose, inverse, conjugate operation, respectively. \(D_n(\cdot)\) is to extract the \(n-th\) row of its matrix argument and rebuild a diagonal matrix out of it. \(\hat{X}\) means estimated value of the matrix \(X\). \(\text{diag}(\cdot)\) is to build up a diagonal matrix which the elements in the brackets take up the diagonal positions. \(\|\cdot\|\) means Frobenius norm of a matrix or vector.

**Data Model**

We consider the base station is equipped with \(M\) and \(N\) antenna elements in X-axis and Y-axis, which are arranged with half-wavelength spacing between adjacent antennas as Figure 1 shows. There exist \(K\) uncorrelated targets in far field impinging on the antenna array.

The targets emit uncorrelated narrow band signals from the direction \(\Theta_1, \Theta_2, \cdots, \Theta_K\), where \(\Theta_i = (\theta_i, \phi_i)\), \(i = 1, 2, \cdots, K\). \(\theta_i\) and \(\phi_i\) are the elevation and azimuth angle of the \(i-th\) incident signal. The received signal of the BS can be expressed as [11, 13]: 

\[
X_{H}S_{N} = \sum_{k=1}^{K} S_s^{*}H_{k}
\]

is the transmitted signal of \(K\) targets. \(L\) is the number of snapshots. \(H\) is the channel matrix. \(N\) is a \(NL\times NL\) matrix, whose elements are uncorrelated zero mean circularly symmetric complex Gaussian noise. \(X\) is the received matrix whose dimension is also \(NL\times NL\). \(H\) means the channel matrix and we divided it into two parts, as \(H = A(\Theta)\Lambda\). \(\Lambda\) is the directional matrix, \(\Lambda\) is the path attenuation coefficient. We assume the source signal from targets arrive the BS through signal path and every path has one specific complex-valued attenuation coefficient on the basis of respective environment.

In rectangular antenna arrays, the directional vectors on the x-axis and y-axis are

\[
a_{x}(\Theta) = \left[1 \ e^{j/2\pi \sin \theta \cos \phi / \lambda} \ \cdots \ e^{j/2\pi (M-1) \sin \theta \cos \phi / \lambda} \right]^T \quad \text{and} \quad a_{y}(\Theta) = \left[1 \ e^{j/2\pi \sin \theta \sin \theta / \lambda} \ \cdots \ e^{j/2\pi (N-1) \sin \theta \sin \theta / \lambda} \right]^T
\]

. The directional matrix on the x-axis and y-axis can be written as

\[
A_x = \left[a_x(\Theta_1) \ a_x(\Theta_2) \ \cdots \ a_x(\Theta_K)\right] \quad \text{and} \quad A_y = \left[a_y(\Theta_1) \ a_y(\Theta_2) \ \cdots \ a_y(\Theta_K)\right].
\]

Hence, the antenna array directional matrix can be expressed as

\[
A = \left[\begin{array}{c}
A_xD_1(A_x) ; A_xD_2(A_x) ; \cdots ; A_xD_N(A_x)
\end{array}\right] \in \mathbb{C}^{MN \times K}.
\]

(1)
DOA and Channel Estimation Algorithm Based on PASTd

Signal Subspace Tracking Algorithm Based on PASTd

In traditional DOA estimation algorithms, such as ESPRIT and MUSIC, we calculate the covariance matrix of the received data at first, and then make eigenvalues decomposition. In this paper, we introduce the subspace tracking method in [4] to obtain the signal subspace. And then, we continue to solve the azimuth and elevation angles as the same as ESPRIT.

We define the cost function as Eq.2. According to [4], the signal subspace can be obtained by minimizing the cost function.

\[
J(W(n)) = \sum_{i=1}^{n} \beta^{n-i} \| X(i) - W(n)W^H(n)X(i) \|^2 = \text{tr} [R(n)] - 2\text{tr} [W^H(n)R(n)W(n)] + \text{tr} [W^H(n)R(n)W(n)W^H(n)W(n)],
\]

where \( 0 < \beta \leq 1 \) is the forgetting factor. \( n \) represents the \( n \)-th snapshot.

We have known \( R(n) = \mathbb{E}[X(n)X^H(n)] \), and \( R(n) \) is the covariance matrix and we replace it with

\[
R(n) = \sum_{i=1}^{n} \beta^{n-i} X(i)X^H(i) = \beta R(n-1) + X(n)X^H(n).
\]

In [4], it has been proved that \( W(n) \) will be close to the eigenvectors of signal subspace in \( R(n) \). Then, we use recursive least squares (RLS) methods to track the signal subspace efficiently. The approximation of \( W(n) \) with \( W(n-1) \) works well for stationary or slowly varying signals.

The elaborate steps of PASTd algorithm to obtain signal subspace are listed as follows:

Step 1: initialize eigenvalue \( d_i(0) \) and eigenvector \( W_i(0) \).

Step 2: for \( n = 1, 2, \cdots, L \) ( \( L \) is gross snapshots), the received signal of every source is \( x_i(n) = X(n) \).

Step 3: for \( i = 1, 2, \cdots, K \), the projection of received signal is represented as:

\[
y_i(n) = W^H_i(n-1)x_i(n), \]

update eigenvalue as:

\[
d_i(n) = \beta d_i(n-1) + |y_i(n)|^2,
\]

update eigenvector as:

\[
W_i(n) = W_i(n-1) + [x_i(n) - W_i(n-1)y_i(n)][y_i^*(n)/d_i(n)],
\]

update received signal of every source as:

\[
x_{i+1}(n) = x_i(n) - w_i(n)y_i(n).
\]

Repeat step 3 after \( i = i + 1 \).

Step 4: let \( n = n + 1 \), and make eigenvalues and eigenvectors update after every iteration. Finally, \( W_i(L) \) could be treated as the signal subspace, and name it \( E_s \).

DOA Estimation

The signal subspace obtained above is utilized in ESPRIT, We define \( E_x = E_x(1 : N(M-1):) \), \( E_y = E_y(N+1 : NM:) \), and can get

\[
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix} = \begin{bmatrix}
AT \\
A\Phi_y T
\end{bmatrix},
\]

where \( E_x(1 : N(M-1):) \) represents the 1st to \( N(M-1) \) rows of \( E_x \), therefore, we can obtain \( E_y = E_yT^{-1}\Phi_y T = E_x\psi_y \), where \( \psi_y = T^{-1}\Phi_y T \). until now, \( E_x \) and \( E_y \) can span similar subspace. And the diagonal elements of \( \Phi_y \) are the eigenvalues of \( \psi_y \). \( \psi_y = E_x^TE_y \) can been acquired using LS method.

Conduct eigenvalues decomposition to \( \psi_y \), get \( r_y = [\sin\phi_1\sin\theta_1, \sin\phi_2\sin\theta_2, \cdots, \sin\phi_k\sin\theta_k] \).

Reconstruct signal subspace,

\[
E_x^\prime = [A_1D_y(A_1); A_2D_y(A_2); \cdots; A_yD_y(A_y)]T.
\]
Use similar method as above, we can obtain $\Phi_x = \text{diag}(\exp(j2\pi\cos\phi_1\sin\theta_1/\lambda), \cdots, \exp(j2\pi\cos\phi_\kappa\sin\theta_\kappa/\lambda))$
and $r_x = [\cos\phi_1\sin\theta_1, \cos\phi_2\sin\theta_2, \cdots, \cos\phi_\kappa\sin\theta_\kappa]$. 

After pairing, we can get final angular estimated values. Angles can be calculated as $[\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_\kappa] = \arcsin\sqrt{\hat{r}_x \hat{r}_x} \text{ and } [\hat{\phi}_1, \hat{\phi}_2, \cdots, \hat{\phi}_\kappa] = \arctan(\hat{r}_x / \hat{r}_x)$, where $\hat{r}_x$ and $\hat{r}_y$ are the estimated values of $r_x$ and $r_y$ when adding gaussian white noise in the channel. Until now, we have estimated the angular information through making use of PASTd to obtain signal subspace. In traditional ESPRIT, we use $R_x = XX^H / L$ and $[EV, D] = \text{eig}(R_x)$ to obtain $E_x$. $R_x$ is the covariance matrix of received signal. \text{eig} means conducting eigenvalue decomposition. $D$ is the diagonal matrix whose diagonal elements are eigenvalues of $R_x$. The signal subspace can be obtained from the eigenvectors $EV$. Comparing to ESPRIT, we find the proposed method has lower computational load because its linear complexity superposition rather than nonlinear one when solve subspace in ESPRIT. The complexity of this traditional method to get $E_x$ is $O((MN)^2L+(MN)^3)$. However, the complexity of PASTd is $O(KL(4MN+2))$. In Fig.2, the complexity comparison is listed in the form of bar graph. We keep other parameters constant except $M$. It’s obvious that the low-complexity PASTd performs much better than ESPRIT, especially when $M$ becomes bigger and bigger.

**Channel Estimation**

The obtained angles in last section can be used in channel estimation. After then, we utilize LS method to get the path attenuation coefficient as Eq.6.

For estimating the attenuation, some pilots are used and known to the BS. We replace the signal source with the pilot, expressed as $P$. The length of pilots is $l_p$. Also, through the DOA estimation above, the BS is able to achieve $A$. We can calculate that


if ignore the influence of noise here, we can get $A \approx A^* Y P^r$. At last, $[\hat{\lambda}_1, \hat{\lambda}_2, \cdots, \hat{\lambda}_K] = \text{diag}(\Lambda)$. It’s necessary to declare that the length of pilots must be longer than the number of users to meet the requirement that $P$ has full row rank.

Hence, the channel matrix estimation can be expressed as $\hat{H} = A(\hat{\Theta}) \cdot \hat{\Lambda}$. Finally, the estimated $A$ and the attenuation coefficients are combined to produce channel matrix.

**Simulation Results**

Eq.7 represents the root-mean-square error (RMSE) of DOA estimation. The received array has $8 \times 8$ antennas whose spacing distance is half wavelength, that is $d = \lambda / 2$.

$$\text{RMSE} = \frac{1}{K} \sum_{k=1}^K \frac{1}{Q} \sum_{q=1}^Q \left[ (\hat{\alpha}_{k,q} - \alpha)^2 + (\hat{\theta}_{k,q} - \theta)^2 \right],$$ (7)

where $\hat{\alpha}_{k,q}$ and $\hat{\theta}_{k,q}$ are the $k$-th target’s estimated values of $q$-th Monte-Carlo simulation. $Q$ is the times of Monte Carlo trail. When estimating the attenuation by pilots, mean-square error (MSE) is used to measure the channel estimation and express it as

$$\text{MSE} = \frac{1}{MN \cdot K} \left\| H - \hat{H} \right\|^2.$$ (8)
Simulation 1: Fig.3 shows the performance of ESPRIT and PASTd on DOA estimation with $K = 2$, $L=200$, $Q = 500$, and $\beta=0.99$. It verifies that our algorithm can also have accurate estimated values just a little worse than ESPRIT.

Simulation 2: Fig.4 is the contrastive curve of LS [12], ESPRIT and our algorithm about channel estimation. Hadamard matrix is used to produce pilot sequences. By the way, the length of pilots must be longer than $K$. It's obvious the channel estimation based on angular information is better than traditional LS algorithm. And our algorithm's performance is also satisfactory.

Simulation 3: In view of the performance of PASTd in DOA and channel estimation, we have done following simulations. Firstly, we assume that there are two unrelated targets in far field whose angles are $(10^\circ,10^\circ)$ and $(20^\circ,20^\circ)$. In Fig.5 When the forgetting factor getting bigger, RMSE becomes lower. The reason is that $\beta$ represents how much the past snapshots will reserve and have an influence on present results.

Simulation 4: Fig.6 show channel estimation performance of our algorithm with $K=2$, $M=8$, and different $N$. From them, the performances are improved with the number of receiving antennas increasing.

Summary

In this paper, we exploit PASTd to obtain the signal subspace and then combine the idea of ESPRIT algorithm for DOA estimation. Moreover, the estimated angular values are utilized to estimate the channel state information. Because our algorithm make use of orthogonal pilots to estimate path attenuation coefficients, lots of transmitted resource for channel estimation will be saved which makes a big difference for actual communication system. Through the simulations above, our algorithm has lower complexity and similar performance as ESPRIT without much computational cost.

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