UAV Detection with S Transform of \textit{t-student} Window

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\textbf{Keywords:} Micro-motion target, S Transform, Short-time fourier transform, Pseudo wigner-ville transform, T-student.

\textbf{Abstract.} So far, the detection of helicopters, especially micro helicopters, has been widely studied, while few works are available on detection of rotor unmanned aerial vehicle (UAV). Due to its low flight height, small RCS and the complicated urban environment, the traditional methods are unable to achieve effective detection of rotor UAV. In this paper, we present a novel S transform with improved window based on \textit{t-student} to deal with this problem. The simulation and field experiments all show that the proposed S transform can suppress Gaussian noise effectively, and obtain the clear time-frequency distribution of the micro-motion target. Thus, the proposed S transform provides some references for detecting the micro-motion target with small RCS and small rotation amplitude such as the rotor UAV.

\textbf{Introduction}

As known, the UAV has been widely used in social life nowadays. The UAV is challenging the public safety because it could be transformed into offensive weapons. So it’s extremely urgent and important to detect and monitor the UAV. However, the conventional detection methods can’t effectively detect the UAV because of the complicated urban environment, its low flight height, small RCS, but great mobility. Time-frequency analysis method was introduced to detect the micro-motion target by V.C. Chen\cite{1}, which provides a new idea for the detection of rotor UAV. So far, there are few research on micro-motion targets such as rotor UAV. Most research on the detection of micro-motion targets aims at large helicopters, ballistic missiles\cite{2,3,4}, etc. Liu W Y\cite{5} presents the auto term window method based on Wigner-Ville distribution(WVD), which effectively extracts the rotating structure of targets. But the cross-term is still disturbed. Li Jingqing\cite{6} proposes the generalized S-Wigner distribution (GSWD) method to detect the ballistic targets. However, his method isn’t suitable for detecting rotor UAV because of the intense time-frequency resolution change of GSWD.

In order to solve the above problems and detect the rotor UAV more effectively, the S transform with \textit{t-student} window is presented in this paper. The simulation and field experiments all show that our approach is effective to suppress Gaussian white noise and obtains clear time-frequency distribution of the echo of micro-motion targets. Our approach could be considered to detect micro-motion targets with small RCS and small rotation amplitude such as the rotor UAV.

\textbf{Echo Signal Model of UAV}

In the optical zone, the EM scattering characteristics of the target can be described by the scattering center model\cite{7}. Generally, the radar target scattering center of the rotor of the UAV is anisotropic due to its special structure and the incident angle of radar wave. The change of the rotor scattering intensity will modulate the echo of the rotor during rotation. Because the size of the rotor is much smaller than the radar resolution, geometrical diffraction theory is adopted\cite{8}.
As shown in Figure 1, the cylinder replaces the rotor of the UAV to simplify the geometry. If the radar transmits single-frequency continuous wave, and the frequency is $f_0$. The rotation angular frequency of the cylinder is $\omega_0$. The coordinate of the center point of the cylinder is $(u, v, w)$. The LOS angle is referred as the angle between LOS and the cylinder center line. The LOS angle of the cylinder changes as sinusoidal function approximately. So, according to GTD model, the scattering intensity of the rotor of UAV can be expressed as the following model[4]:

$$\sigma(t) = \sigma_0 \{ \rho + \exp[\phi_0 \sin(\omega_0 t + \eta)] \}.$$  \hfill (1)

Where $\sigma_0$, $\rho$, $B$ and $\eta$ are constants, and depend on the characteristics of scattering center. Considering the anisotropy of scattering target, the echo signal of the rotor of UAV can be expressed as follows:

$$s(t) = \sum_{l=1}^{L} \sigma_0 l \{ \rho l + \exp[B l \sin(\alpha_{0l} t + \eta)] \} \exp\left\{ \frac{4\pi f_0}{c} [\sigma_0 l + A_{0l} \sin(\alpha_{0l} t + \phi_l)] \right\} \exp\left\{ j2\pi f_0 \frac{2\eta(t)}{c} \right\}. $$  \hfill (2)

Where $L$ represents the number of scatter centers. $\sigma_0$, $\rho$, $B$ and $\eta$ represent the scattering characteristics of scattering center $l$. $r_0^l$ is the initial distance from radar to scattering center. $A_{0l}^i$ is the rotation radius of scattering center $l$. $\phi_l$ is the initial phase of scattering center $l$. $c$ is the light velocity. Thus, the echo of the rotor of UAV is the multi-component amplitude modulation-frequency modulation (AM-FM) signal. The AM and FM of the each signal component is sinusoidal modulation, and the modulation frequency is the rotation frequency.

**S Transform with the Improved Window**

S transform[9,10] provides frequency-dependent resolutions and retains the absolute phase of time series component. Furthermore, it provides time-frequency representation of the signal so that the components of the signal can be isolated and processed independently in the time-frequency plane. The generalized S transform of continues time series $h(t)$ is [11]:

$$S(\tau, f, p) = \int_{-\infty}^{\infty} h(t) \omega(t - \tau, f, p) \exp(-j2\pi ft) dt.$$  \hfill (3)

According to the convolution theory, the S transform can be written as a function of Fourier transform of time series, as follows:

$$S(\tau, f, p) = \int_{-\infty}^{\infty} H(\alpha + f) W(\alpha, f, p) \exp(j2\pi ft) d\alpha.$$  \hfill (4)
Where $H$ and $W$ are the Fourier transform of $f$ and $t$, respectively. Typically, Gaussian function is applied as the modulating window in the S transform. The Gaussian window in the frequency domain $W_{GS}$ is:

$$W_{GS(a,f,|\gamma_{GS}|)} = \exp\left(-\frac{2\pi^2\alpha^2\gamma^2_{GS}}{f^2}\right).$$

(5)

Where $\gamma_{GS}$ is the adjustable parameter, which has been defined to tune the time and frequency resolutions of the S transform. From the above formula we clearly know that the Gaussian window has narrow width at the low frequency, which means S transform has high frequency resolution and low time frequency resolution at the same time. Because of the quick and intense variations of the Gaussian window over frequency, the S transform with Gaussian window is not appropriate in the detection of rotor UAV. Thus, a window with gradual changes over frequency may improve the S transform and make it suitable for the detection of rotor UAV.

$T$-student distribution[11] has more gradual changes over frequency in comparison with sharp changes of Gaussian window. Thus, we propose the $t$-student based window $W_{ts}$ over $x$ with the freedom degree of $\nu$ for the S transform as follows:

$$W_{ts}(x,v,p) = F(x|v) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\nu\pi}} \frac{1}{1+\frac{x^2}{v}}.$$

(6)

Where $\Gamma(\cdot)$ is the Gamma function. In order to obtain a more uniform time and frequency resolutions, we applied a window with $t$-student distribution over $\alpha^2/f$ with one degree of freedom $F(\alpha^2/f|\nu)$. Furthermore, an adjustable parameter $\gamma_{Pr}$ is added to the proposed window to tune the time and frequency resolutions of the S transform. The S transform with the improved windows is as follows:

$$W_{ts}(\alpha,f,|\gamma_{Pr}|) = \int_{-\infty}^{+\infty} H(\alpha+f) \frac{1}{1+\left(\frac{\alpha^2}{f}\gamma_{Pr}\right)} \exp(j2\pi\alpha\tau)d\alpha.$$

(7)

The amplitude of the proposed window scales with the square root of frequency unlike other windows previously used in the S transform in which the amplitude is scaled linearly with frequency. The width of the all windows scales with the inverse of their amplitude. Variation of the proposed window with the square root of frequency instead of linear scaling provides more uniform spectral field because of more gradual changes in the window.

**Simulation and Experiment**

In this section, the S transform with the proposed window is compared with other conventional time-frequency distribution methods such as STFT and PWVD. The simulation setup is as shown in Figure 1, we set: $f_c=18$GHz, PRF=50kHz, and SNR=20dB, $\omega_0=200\pi$. In general, we set $L=2$. For the scattering center $l$, $\sigma_{0}=0.01$, $A_{0i}=0.05m$, $(u,v,w)=(10,40,0)$.

The STFT, PWVD and the proposed S transform distribution of the echo of micro-motion targets are shown in Figure 2.
From the simulation results, we clearly know that the Gaussian noise has a serious impact on STFT distribution. PWVD distribution is disturbed by the noise and self-cross terms. So we can’t recognize the micro-motion curve from STFT and PWVD. However, S transform with the improved window performs a better ability to suppress Gaussian noise and has no self-cross terms. We can recognize the micro-motion curve and get the target information more clearly.

To verify the performance of the S transform with the improved window further, we perform a field test of a plastic fan in the anechoic chamber. The experiment setup is shown in Figure 3, two standard horn antennas 1 and 2 are used as the transmitting and receiving antennas respectively. The distance of two antennas is 0.3 meter and they are all aimed at the plastic fan. The plastic fan is placed 1 meter away from two antennas, and they are at the same horizontal height. The blades plane of fan is facing the horn antennas with tilt angle of about 30 degrees. The transmission signal is the single-frequency signal and the frequency is 16GHz. The sampling frequency is 38kHz.
Figure 4 shows the comparison results of applying the STFT, PWVD and S transform with the improved window. The comparison results show that the echo of the plastic fan doesn’t behave as the sinusoidal frequency modulation signal, and it behaves like the periodical discrete ridges. The reason is that the phase of the echo changes as the sinusoidal frequency modulation signal with a small amplitude, but the amplitude of the echo changes as the sine-like modulation with great amplitude. So the time-frequency distribution of the echo appears like the discrete and concentrated ridges. Just as the simulation shows, the STFT or PWVD distribution still could not achieve a satisfactory performance in the field experiment. In contrast, S transform with the improved window performs a certain ability to suppress Gaussian noise compared with the STFT distribution and has no cross-terms compared with PWVD. Furthermore, we can estimate the rotational frequency and other parameters of the plastic fan from the S transform distribution of the echo in the field experiment. There is a good correlation between the simulation and experiment results.

Summary

This paper has proposed a novel detection method of micro-motion targets. The proposed method adopts the $t$-student based window to replace the Gaussian window. The $t$-student based window changes the width and amplitude with the square root of frequency. This relatively gradual variation of the proposed window improves the time resolution of low-frequency components as well as the frequency resolution of high frequencies. Compared with STFT and PWVD, the simulation and field experiments all show that the proposed S transform can suppress Gaussian noise effectively, improve signal to noise ratio and obtain clear time-frequency distribution of the micro-motion target. Besides, the proposed S transform doesn’t produce cross-terms. So the proposed S transform could be considered for detecting the micro-motion target with small RCS and small rotation amplitude such as the rotor UAV and improve the detection range of UAV to some extent. However, our proposed method may fail to detect the UAV in the strong clutter environment. Thus, we attempt to use other clutter suppression method to improve the detection in the future.
Acknowledgements

This work was jointly supported by the National Science Foundation of China under Grant (61372160). Besides, the authors are grateful to Song Xiaoji for supporting in experiments. Thanks for the helpful discussions to Chen Qiao and Lu Jianrong.

References


