A Sliding Mode Guidance Law with ESO for Near Space Interceptor

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Abstract. Considering the autopilot dynamic characteristics of the near space interceptor and the target maneuver, a novel sliding mode guidance law is proposed based on the extended state observer (ESO). Firstly, the ESO is constructed to estimate the target acceleration of the disturbances for the guidance system, and the disturbance estimated values are employed as the feed-forward compensation term to obtain the robustness of the target maneuver and system disturbances. Then, a novel sliding mode guidance law based on the ESO is given. Finally, simulation results for the near space interceptor guidance system show that the proposed guidance law can not only overcome the effect of the autopilot dynamic delay and target maneuver, but also ensure the accurate guidance results.

Introduction

To deal with the threat of the near space air-breathing hypersonic vehicles (NSHV), near space interceptor (NSI) with aerodynamic fins and reaction jets is a kind of effective defense scheme. To implement the objective of hitting the NSHV target directly, guidance law design is the most important technology in the near space interception. Moreover, the model uncertainties and disturbances, which mainly include the variations of the target’s maneuvers and changing engagement conditions, make the NSI guidance system design more difficult [1].

To intercept the larger maneuvering target, some advanced control methods have also been applied to the guidance law, such nonlinear $H_\infty$ guidance law [2,3], sliding mode guidance (SMG) law [4], and so on. However, the above methods dealt with the influence of the target maneuvers at the price of sacrificing the normal guidance performance. In [2], the guidance law design is formulated as a nonlinear $H_\infty$ disturbance attenuation problem, in which the target accelerations are regarded as unpredictable disturbances. In particular, the nonlinear $H_\infty$ control problem relies on the solution of the Hamilton-Jacobi-Isaacs (HJI) equation, which is a partial differential equation (PDE) and notoriously difficult to solve both numerically and analytically. SMG is an efficient method to deal with the influence of the target maneuver and parameter uncertainties, but chattering is an unavoidable problem of engineering application [4]. The saturation function and boundary layer method are often used to alleviate the chattering at the price of the disturbance rejection performance.

Extended state observer-based control (ESOBC) has drawn much attention in the past few years [5,6]. In the framework of ESOBC, a baseline controller is firstly designed under the assumption that there are no system disturbances, such as sliding mode control, optimal control, and so on; then, the feed-forward compensation term is introduced to remove the influences of disturbances, which can be estimated by the proposed extended state observer (ESO).

Motivated by the aforementioned considerations, this paper dealt with the guidance law design problem for the NSI based on the sliding mode control and ESO. Simulation results also show the effectiveness of the proposed design method.
Guidance Model Description of the NSI

Define the following variables \( x_1 = q_e \), \( x_2 = \dot{q}_e \), \( x_3 = a_{me} \), \( x_4 = \dot{a}_{me} \), \( u_e = a_{ue} \), \( x_{p1} = q_{p} \), \( x_{p2} = \dot{q}_{p} \), \( x_{p3} = a_{mp} \), \( x_{p4} = \dot{a}_{mp} \), the guidance system of the NSI can be written as the following form:

\[
\begin{aligned}
    \dot{x}_1 &= 2 \frac{\dot{r}}{r} x_1 - x_1^2 \sin x_1 \cos x_1 - \frac{x_1}{r} a_n + \frac{a_n}{r} \\
    \dot{x}_2 &= 2 \frac{\dot{r}}{r} x_2 + 2 x_1 x_2 \tan x_1 + \frac{x_{p3}}{r \cos x_1} - \frac{a_{p}}{r \cos x_1} \\
    \dot{x}_3 &= x_3 \\
    \dot{x}_4 &= -2 \zeta \omega_n x_4 - \alpha_n^2 x_3 + \omega_n^2 u_e + d_x
\end{aligned}
\]

(1)

\[
\begin{aligned}
    \dot{x}_{p1} &= 2 \frac{\dot{r}}{r} x_{p1} - x_{p1}^2 \sin x_{p1} \cos x_{p1} - \frac{x_{p1}}{r} a_{p} + \frac{a_{p}}{r} \\
    \dot{x}_{p2} &= 2 \frac{\dot{r}}{r} x_{p2} + 2 x_{p1} x_{p2} \tan x_{p1} + \frac{x_{p3}}{r \cos x_{p1}} - \frac{a_{p}}{r \cos x_{p1}} \\
    \dot{x}_{p3} &= x_{p3} \\
    \dot{x}_{p4} &= -2 \zeta \omega_{p} x_{p4} - \alpha_{p}^2 x_{p3} + \omega_{p}^2 u_{p} + d_{p}
\end{aligned}
\]

(2)

where \( q_e \) and \( q_{p} \) are the azimuth angle and elevation angle in the LOS coordinate system, respectively; \( r \) is the relative distance between the interceptor and target; \( [a_{me}, a_{ne}, a_{mp}]^T \) and \( [a_{ne}, a_{a_{p}}, a_{p}]^T \) are the interceptor and target accelerations in the LOS coordinate system; \( a_{me}^* \) and \( a_{mp}^* \) denote the acceleration command of the NSI in the vertical and lateral plane, respectively; \( \zeta \) and \( \omega_{a} \) denote the damping ratio and natural frequency of the NSI autopilot, respectively; \( d_{x} \) and \( d_{p} \) are the disturbances and parameter uncertainties in the autopilot loop.

Guidance Law Design

In this section, a novel guidance law is developed for the time-varying systems Eq.1 and Eq.2 based on the sliding mode control and ESO.

Guidance Law Design for the Longitudinal Plane

Define the variable \( v_e = \dot{r} q_e \), we have from Eq. 1

\[
\dot{v}_e = -\frac{v_e}{r} - \frac{v_{e}^2 \tan q_e}{r} + a_{n} - x_3
\]

(3)

Then, the following ESO is proposed to estimate the target acceleration \( a_{n} \):

\[
\begin{aligned}
    \dot{e}_{21} &= z_{21} - v_e \rangle \varepsilon_{22} = z_{22} - a_{n} \\
    \dot{z}_{21} &= 2 \frac{\dot{r}}{r} v_e + \frac{\dot{v}_e}{r} \tan q_e \rangle r - x_3, \quad e_{21} \text{ and } e_{22} \text{ are the estimation errors of the ESO, } \\
    \dot{e}_{22} &= \mu_{21} e_{21} + f_{e2}
\end{aligned}
\]

(4)

where \( f_{e2} = -v_e v_e \rangle r - (v_e^2 \tan q_e) \rangle r - x_3 \), \( e_{21} \) and \( e_{22} \) are the estimation errors of the ESO, \( \mu_{21} \) and \( \mu_{22} \) are the observer gain coefficients. The function \( \text{fall}(\cdot) \) is defined as the following form:

\[
\text{fall}(\varepsilon_{21, \alpha_{e}, \delta_{2}}) = \begin{cases} 
    \varepsilon_{21}^{\mu_{a}+1} \text{sgn}(\varepsilon_{21}) & |\varepsilon_{21}| > \delta_{2} \\
    \varepsilon_{21} / \delta_{2}^{\mu_{a} - 1} & |\varepsilon_{21}| \leq \delta_{2}
\end{cases}
\]

(5)

with \( 0 < \alpha_{e, a} < 1 \) and \( 0 < \delta_{e} < 1 \).

Based on the same method, the following ESO is proposed to estimate the disturbances in the autopilot loop \( d_{x} \):
where $f_{e} = -2\zeta \alpha _{e} x_{i_{e}} - \alpha _{e}^{2} x_{i_{e}}^{2} + \rho _{e}$, $e_{i_{e}}$ and $e_{i_{e}^{2}}$ are the estimation errors. The function $fal(\cdot)$ is defined as the following form:

$$
fal(e_{i_{e}}, \alpha _{e}, \delta _{e}) = \begin{cases} 
[e_{i_{e}}]^{\alpha _{e}} \text{sgn}(e_{i_{e}}) & |e_{i_{e}}| > \delta _{e} \\
\frac{e_{i_{e}}}{\alpha _{e}} & |e_{i_{e}}| \leq \delta _{e} 
\end{cases}
$$

Based on the system Eq.1 and the ESO Eq.4 and Eq.7, we consider the following guidance law:

**Step 1:** Define the first error surface as follows:

$$s_{e_{2}} = x_{e_{2}}$$

Then, the following virtual control $\dot{s}_{e_{2}}$ is selected to make $s_{e_{2}}$ to converge to zero:

$$\dot{s}_{e_{2}} = -\frac{2\ddot{r}}{r} x_{e_{2}} - x_{e_{2}}^{2} \sin x_{e_{2}} \cos x_{e_{2}} - \frac{x_{e_{2}}^{3}}{r} + \frac{a_{e}}{r}$$

**Step 2:** Denote the second error surface as follows:

$$s_{e_{3}} = x_{e_{3}} - \overline{x}_{e_{3}}$$

Then, the following virtual control $\dot{s}_{e_{3}}$ is selected to make $s_{e_{3}}$ to converge to zero:

$$\dot{s}_{e_{3}} = -k_{e_{3}} s_{e_{3}} + \overline{x}_{e_{3}}$$

**Step 3:** Denote the third error surface as follows:

$$s_{e_{4}} = x_{e_{4}} - \overline{x}_{e_{4}}$$

Then, we can design the following actual control to make $s_{e_{4}}$ to converge to zero:

$$u_{e} = \alpha _{e}^{2}[2\zeta \alpha _{e} x_{e_{4}} + \alpha _{e}^{2} x_{i_{e}}^{2} - \dot{\delta}_{e} x_{e_{4}} + \dot{x}_{e_{4}}^{2} - k_{4} s_{e_{4}} + k_{5} x_{e_{4}}^{2}] \text{sgn}(s_{e_{4}})]$$

where $k_{4} > 0$, $k_{5} > 0$, $0 < \lambda _{e} < 1$.

**Guidance Law Design for the Lateral Plane**

Define the variable $v_{p} = r \dot{y}_{p} \cos q_{e}$, we have from Eq.2

$$\dot{v}_{p} = \frac{v_{p} \dot{v}_{p}}{r} + \frac{v_{p} v_{p} \tan q_{e}}{r} - \dot{a}_{p} + x_{p3}$$
Then, the following ESO is proposed to estimate the target acceleration $a_{\mu}$:

$$\begin{align*}
\dot{e}_{p21} &= z_{p21} - v_{\mu}, \quad e_{p22} = -z_{p22} - a_{\mu} \\
\dot{\delta}_{p21} &= z_{p21} - \mu_{p21} e_{p21} + f_{p2} \\
\dot{\delta}_{p22} &= -\mu_{p22} \text{fal}(e_{p21}, \alpha_{p2}, \delta_{p2})
\end{align*}$$

(20)

where $f_{p2} = -v_{\mu} v_{\mu} / r + (v_{\mu} v_{\mu} \tan q_{\mu}) / r + x_{p3}$. The function $\text{fal}(\cdot)$ is defined as the following form:

$$\text{fal}(e_{p21}, \alpha_{p2}, \delta_{p2}) = \begin{cases} 
|e_{p21}| / \alpha_{p2} & |e_{p21}| > \delta_{p2} \\
|e_{p21}| / \delta_{p2} & |e_{p21}| \leq \delta_{p2}
\end{cases}$$

(21)

Based on the same method, the following ESO is proposed to estimate the disturbances in the autopilot loop $d_{p4}$:

$$\begin{align*}
\dot{e}_{p41} &= z_{p41} - x_{p4}, \quad e_{p42} = z_{p42} - d_{p4} \\
\dot{\delta}_{p41} &= z_{p42} - \mu_{p41} e_{p41} + f_{p4} \\
\dot{\delta}_{p42} &= -\mu_{p42} \text{fal}(e_{p41}, \alpha_{p4}, \delta_{p4})
\end{align*}$$

(22)

where $f_{p4} = -2\dot{z} \omega x_{p4} - \alpha_n^2 x_{p4} + \alpha_n^2 u_{\mu}$. The function $\text{fal}(\cdot)$ is defined as the following form:

$$\text{fal}(e_{p41}, \alpha_{p4}, \delta_{p4}) = \begin{cases} 
|e_{p41}| / \alpha_{p4} & |e_{p41}| > \delta_{p4} \\
|e_{p41}| / \delta_{p4} & |e_{p41}| \leq \delta_{p4}
\end{cases}$$

(23)

Based on the system Eq.2 and the ESO Eq.21 and Eq.23, we consider the following guidance law:

$$\begin{align*}
s_{p2} &= x_{p2} \\
x_{p3} &= 2r_{\mu} \cos x_{x3} - 2r_{x2}x_{p2} \sin x_{x3} + \dot{a}_{\mu} - k_{p2} s_{p2} \\
\tau_{p3} &= \ddot{x}_{p3}, \quad x_{p3}(0) = x_{p3}^{*}(0) \\
\tau_{p4} &= -k_{p3} \ddot{x}_{p3} + \ddot{x}_{p3} \\
\ddot{x}_{p4} &= \ddot{x}_{p4} + \ddot{x}_{p4}^{*}(0) = x_{p4}^{*}(0) \\
\dot{s}_{p4} &= x_{p4} - \ddot{x}_{p4} \\
u_{\mu} &= \alpha_n^2 \left[ 2\dot{z} \omega_{z} x_{p4} + \alpha_n^2 x_{p3} - \dot{d}_{p4} + \ddot{x}_{p4} - k_{p4} s_{p4} - k_{p5} |s_{p4}|^{\gamma} \text{sgn}(s_{p4}) \right]
\end{align*}$$

(24)

Simulation Results

In this section, some numerical simulations for the NSI guidance system will be given to demonstrate the effectiveness of the proposed guidance scheme. The initial conditions of the NSI and target in inertial coordinate system are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{\text{in}}$</td>
<td>0 [m]</td>
<td>$V_{\text{in}}$</td>
<td>1500 [m/s]</td>
<td>$x_0$</td>
<td>15204.6 [m]</td>
<td>$V_0$</td>
<td>1700 [m/s]</td>
</tr>
<tr>
<td>$y_{\text{in}}$</td>
<td>0 [m]</td>
<td>$u_{\text{in}}$</td>
<td>30 [°]</td>
<td>$y_0$</td>
<td>6840.4 [m]</td>
<td>$\theta_0$</td>
<td>−10 [°]</td>
</tr>
<tr>
<td>$z_{\text{in}}$</td>
<td>0 [m]</td>
<td>$\psi_{\text{in}}$</td>
<td>−30 [°]</td>
<td>$z_0$</td>
<td>11046.8 [m]</td>
<td>$\psi_0$</td>
<td>140 [°]</td>
</tr>
</tbody>
</table>

The parameters of the ESO are given as follows:

$\mu_{p21} = \mu_{p22} = 50$, $\mu_{p23} = \mu_{p24} = 150$, $\alpha_{p23} = \alpha_{p24} = 0.2$, $\alpha_{p25} = \alpha_{p26} = 0.001$ ,
$\mu_{p41} = \mu_{p42} = 5$, $\mu_{p43} = \mu_{p44} = 200$, $\alpha_{p41} = \alpha_{p42} = 0.2$, $\alpha_{p43} = \alpha_{p44} = 0.01$.

The following interception conditions are selected to test the effectiveness of the proposed algorithm:
Sinusoidal maneuvering target, \( a_x = a_y = 40 \sin(t) \) m/s^2.

And, the disturbances of the autopilot loop are selected as \( d_{x4} = 200 \sin(t) \) and \( d_{y4} = 200 \cos(t) \).

The proposed guidance law based on the sliding mode and ESO (denoted by BSM+ESO) is applied to the NSI guidance system, and the PNG and SMG are also studied for the simulation comparisons. The response curves of LOS angle rate based on the three guidance schemes are shown in Figure 1.

![Figure 1. Curves of the LOS angular rate for given sinusoidal maneuvering target.](image)

Under the same simulation condition, we can obtain that the proposed sliding mode guidance scheme can make the LOS angle rate converge to the steady state value more quickly, and the transient value of the system response is also small.

The simulation result comparisons of the three guidance schemes are shown in Table 2 in terms of miss distance and interception time. We can see from Table 2 that the miss distance for BSM+ESO is minimal.

<table>
<thead>
<tr>
<th>Guidance method</th>
<th>Miss Distance [m]</th>
<th>Interception Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PNG</td>
<td>1.2950</td>
<td>7.022</td>
</tr>
<tr>
<td>SMG</td>
<td>1.1500</td>
<td>7.020</td>
</tr>
<tr>
<td>BSM+ESO</td>
<td>0.1683</td>
<td>7.020</td>
</tr>
</tbody>
</table>

**Conclusion**

In this paper, a novel guidance law is proposed for the NSI based on the sliding mode control and ESO. Firstly, the guidance system with the autopilot dynamic can be divided into the outer loop, middle loop and inner loop. Then, the ESO is developed to estimate the target acceleration of the outer loop and the disturbance of the inner loop, and the estimated values are employed as the feed-forward compensations to remove the influence of the total system disturbances. Finally, simulation results demonstrate that the proposed guidance law can obtain a small miss distance compared with the other guidance schemes.

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