Preliminary Theory of Set DR of Fuzzy Time Series Forecasting Model

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Keywords: Fuzzy time series forecasting model, DR, Difference rate, Prediction function SD(μ), Convergence Theorem.

Abstract. In order to solve the question which the existing fuzzy time series forecasting model prediction accuracy is not high, this paper proposes the fuzzy time series forecasting model based on differential collection of SD. The general elements expressed in SD(μ). Prove: IF a time series forecasting method requires AFER\(\leq B\) (an arbitrary small positive number) and MSE\(\leq C\) (a positive number), then there exist independent variables \(\mu_0 \in (0,1)\), when it uses forecasting model DR for time series prediction model to simulate the problem of history data prediction research, to ensure that the average prediction error rate AFER\(\leq B\) and mean square error MSE\(\leq C\) was established at the same time.

Introduction

Time series forecasting method is an ancient topic nearly hundred years of history, but its application universality makes people constantly explore new prediction model. Therefore, new methods constantly emerging. Particularly In 1993, Song et al[1-3] proposed the first fuzzy time series forecasting model[4] and used it to study the enrollment prediction of Alabama University from 1971 to 1992, which pushed the research of time series forecasting method to a new stage. In 2007, Jilani et al[5] proposed the inverse fuzzy number firstly and built a forecasting model based on it for the study of fuzzy time series model. In 2009, Stevenson et al[6] proposed an improved fuzzy time series forecasting model. Especially in 2012, Saxena et al[5] proposed an improved fuzzy time series forecasting model based on inverse fuzzy number, it is used to study at the University of Alabama at the number of registered prediction problem, the mean square error and the average forecasting error rate of their model are MSE=9169 and AFER=0.3406%, they provided the smallest AFER and MSE than the existing methods (until 2012)[13]. The acquisition of such a high prediction accuracy is inseparable from the concept of inverse fuzzy number. Feng et al[14] and Wang et al[8-13] of the presented model respectively, are also used inverse fuzzy number concept, are all improved model prediction model is proposed for literature[5-7], which Makes respectively applied to the number of registered at the University of Alabama at most have certain to improve prediction accuracy of forecasting problems. For example Feng et al[8] proposed method gets higher prediction accuracy of AFER=0.0099% and MSE=7.

In this paper, it further promote the literature [5-10] prediction model, build a set of fuzzy time series prediction method for the element, it’s called a set of fuzzy time series forecasting model based on difference rate. SFTSFMBDR (The Set of Fuzzy Time Series Forecasting Models Based on the Difference Rate), further simplify the abbreviation for DR and preliminary study of the basic theory of DR. This paper proved Any Accuracy Theorem of DR: When prediction model DR for the time series prediction problem of simulations to predict the historical data, For a given arbitrarily small positive number B and any positive number C, then there exist independent variables \(\mu_0 \in (0,1)\), when it used forecasting model DR for time series prediction model to simulate the problem of history data prediction research, to ensure that the average prediction error rate AFER\(\leq B\) and mean square error MSE\(\leq C\) was established at the same time.
Basic Concepts
This paper uses the main concepts and the main formula shown as below.

Definition 1 For time series prediction problem of history data \( H = \{H_1, H_2, \ldots, H_n\} \). We define the difference as

\[ A_e = H_e - H_{e-1}. \]  

Definition 2 Use the following formula for inspection prediction error: \( K_e - H_e \);

predictive variance: \( (K_e - H_e)^2 \);

MSE (Mean Square Error): \( \text{MSE} = \frac{1}{n} \sum_{e=1}^{n} (K_e - H_e)^2 \);

prediction error rate: \( |K_e - H_e| / H_e \);

AFER (Average Forecasting Error Rate): \( \text{AFER} = \frac{1}{n} \sum_{e=1}^{n} |K_e - H_e| / H_e \)

where \( K_e \) is called the forecast number of \( e \)-year, \( H_e \) is called historical data of \( e \) years.

Preliminary Theoretical Research of Set DR

Prediction Formula SD(\( \mu \))

Definition 3 For difference rate of history data \( A = \{A_1, A_2, A_3, \ldots, A_n\} \). we define inverse fuzzy number as

\[ U = \frac{b+1}{b} \frac{1}{A_{e-1} + A_e}, \]

where \( U \) is the inverse fuzzy number of difference rate of history data \( A \), and constant \( b \in (0,1) \) is called membership number of \( A_{e-1} \).

Definition 4 For each given \( e \in \{3,4,\ldots,n\} \), difference of history data \( A = \{A_2, \ldots, A_n\} \), we define inverse fuzzy number function as:

\[ U_e(\mu) = \frac{\mu+1}{\mu} \frac{1}{A_{e-1} + A_e}, \]  

where \( U_e(\mu) \) is the inverse fuzzy number function of difference of history data \( A \), \( A_{e-1} \) is the difference of history data of \( e-1 \)-year, \( A_e \) is the difference of history data of \( e \)-year, \( \mu \in (0,1) \) is called membership number of \( A_{e-1} \). When \( \mu \) given a specific argument value, inverse fuzzy number function \( U_e(\mu) \) is an inverse fuzzy number.

Definition 5 For each given \( e \in \{3,4,\ldots,n\} \), time series prediction problem of history data \( H = \{H_1, H_2, \ldots, H_n\} \) and difference of history data \( A = \{A_2, A_3, \ldots, A_n\} \), we define the prediction function as

\[ K_e(\mu) = H_{e-1} + U_e(\mu)=H_{e-1} + \frac{\mu+1}{\mu} \frac{1}{A_{e-1} + A_e}, \]

where \( K_e(\mu) \) is the prediction function of history data of \( e \)-year, \( \mu \in (0,1) \) is a independent variables, it also called membership number of the prediction function. Where \( H_{e-1} \) is the history data of \( e-1 \)-year, \( U_e(\mu) \) is the inverse fuzzy number function of \( e \)-year, and \( A_{e-1} \) is the difference of history data of \( e-1 \)-year, \( A_e \) is the difference of history data of \( e \)-year. When given the independent variables of a specific value, the prediction function can be called a prediction formula.
He Application Step of Prediction Formula of SD($\mu$)

When determining the membership number $\mu \in (0,1)$ as a specific numerical value, SD ($\mu$) is a prediction formula. We can apply SD ($\mu$) to research time series analysis. The application procedure is as follows.

i) Establishing historical data table for time series prediction problem;

ii). Establishing theory of domain (include domain of history data H, difference of history data A);

iii). Writing prediction formula SD ($\mu$); iv). Applying SD ($\mu$) to compute the prediction value of history data;

Theorem 1 When determining the membership number $\mu \in (0,1)$ as a specific numerical value, the prediction formula SD ($\mu$) is also a forecasting model of time series. Applying procedure of forecasting model of time series SD ($\mu$) is also the application procedure of the prediction formula SD ($\mu$).

Set SD of Forecasting Model of Fuzzy Time Series

Definition 6 If $\mu$ takes all value in (0,1), we can obtain a system of time series forecasting model SD ($\mu$). The sum of system of time series forecasting model SD ($\mu$) is called set of fuzzy time series forecasting model based on difference, for short, it is written as SFTSFMBDR (The Set of Fuzzy Time Series Forecasting Models Based on the Difference Rate). Its general element is SD ($\mu$). SD ($\mu$) denotes a forecasting formula of fuzzy time series and a fuzzy time series forecasting model.

Remark From definition 6 can know, SD is a collection of time series forecasting model, but inverse fuzzy number function (2) is an important component of prediction function(3), so we call the time series forecasting model as fuzzy time series forecasting model.

Basic Theory of SET SD

Theorem 2 (Convergence Theorem Based on Fuzzy Function of set SD) For time series prediction problem of history data H = {H1, H2, ..., Hn} and difference of history date A= { A2, A3, ..., An}. Then for every $e \in \{3,4,...,n\}$, when $\mu \rightarrow 0$, Then the historical data of the prediction function $K_e(\mu)$ converges to the historical data $H_e$,

$$\lim_{\mu \rightarrow 0} K_e(\mu) = H_e$$

Proof For each $e \in \{3,...,n\}$, according to definition 1 we have

$$A_e = H_e - H_{e-1} \Rightarrow H_e = A_e + H_{e-1}.$$ 

Letting $\mu \rightarrow 0$, we get

$$\lim_{\mu \rightarrow 0} K_e(\mu) = \lim_{\mu \rightarrow 0} (H_{e-1} + U_e(\mu)) = H_e - \lim_{\mu \rightarrow 0} U_e(\mu)$$

$$= H_{e-1} + \lim_{\mu \rightarrow 0} \frac{\mu + 1}{A_{e-1}} = H_{e-1} + \frac{\lim_{\mu \rightarrow 0} (\mu + 1)}{\lim_{\mu \rightarrow 0} (A_{e-1} + 1)} = H_{e-1} + A_e = H_e$$

Theorem 3 (Any Accuracy Theorem of SD) Let H = {H1, H2, ..., Hn} be time series prediction problem of history data, and A= { A2, A3, ..., An} be difference of history date. For a given arbitrarily small positive number B and any positive number C, then for every $\mu \in (0,1)$, when prediction model SD for the time series prediction problem of simulations to predict the historical data, to ensure that the average prediction error rate AFER$ \leq p$ and mean square error MSE$ \leq q$ was established at the same time.
\[ \text{AFER} = \frac{1}{n-2} \sum_{e=3}^{n} |K_e(\mu) - H_e| / H_e < B \quad \text{and} \quad \text{MSE} = \frac{1}{n-2} \sum_{e=3}^{n} (K_e(\mu) - H_e)^2 < C \]

**Proof** It is not difficult to apply the SD prediction function convergence theorem to prove this theorem.

**Reasoning 1** Forecasts of the registered number of historical data for theory domain \( H= \{ H_{1971}=13055, H_{1972}=13563, \ldots, H_{1992}=18876 \}^{[1-3]} \) at the university of Alabama in 1971 ~ 1992, and difference of history date \( A= \{ A_{1972}=508, A_{1973}=304, \ldots, A_{1991}=9, A_{1992}=-461 \} \). Then for every \( e \in \{2,3,\ldots,n\} \), \( B=0.3406\% \), \( C=9169 \). There is independent variables \( \mu_0 \in (0,1) \), when prediction model SD for the time series prediction problem of simulations to predict the historical data, it can ensure that the average prediction error rate AFER \( \leq B \) and mean square error MSE \( \leq C \) was established at the same time.

**Proof** This is the direct inference of Any Accuracy Theorem.

For example, in the study of the number of registered prediction problems at the University of Alabama in 1971 ~ 1992, if required AFER \( < 0.3406\% \) and MSE \( < 9169 \) formed at the same time. For the variable \( \mu_0=0.004 \in (0,1) \), when use the forecast model SD (0.004) of set SD to research this problem, it would get AFER=0.058745\% <0.3406\% and MSE=1520<9169 at the same time.

But the prediction accuracy of prediction model of fuzzy time series forecasting models of the set SDR is not all higher, some of the forecasting model is standard, accuracy is high; There are also many forecasting model is not standard, accuracy is not high. It needs specific conditions to measure if predictive model is standard. Saxena etc. \cite{7} proposed a forecasting model of fuzzy time series in 2012 and get very good results, i.e. Mean Square Error MSE=9169 and AFER=0.3406\% to predict the enrollment number of Alabama University in 1971~1992 year , so those data are used to measure the criterion of good forecasting model of fuzzy time series. With these data as a condition to distinguish if a fuzzy time series forecasting model is standard. Therefore, we establish the following definition.

**Definition 7** A forecasting model of fuzzy time series is called standard, it was noted above that, if we use it to predict the enrollment number of Alabama university in 1971~1992 year, we can get AFER=0.0841\% and MSE=1382, So the SDR (0.004) is also the standard fuzzy time series forecasting model.

**Conclusion**

The structure of each prediction formula of forecast model of set SDR of Fuzzy time series prediction model is simple and convenient. Convergence Theorem Based on Fuzzy Function of set SDR (Theorem 1) is the theory basis of the set SDR of fuzzy time series forecasting model. Any Accuracy Theorem (Theorem 3) preliminary solved the problem of that the existing fuzzy time series forecasting model prediction accuracy is not high. SDR(\( \mu \)) (\( \mu \leq 0.004 \)) is the standard fuzzy time series forecasting model, and it can be used for general research of time series prediction. The task in future will focus on studying how to develop the predicted value of the SDR(\( \mu \)) for the unknown data prediction method.

**Acknowledgement**


**References**


