A Method for the Measurement of Second Heart Sound Split Based on Modified S Transform

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Abstract. The time delay between A2 and P2, which is known as S2 split is reported to have significant diagnostic potential. This paper proposes a target function to optimize the standard S transform, and the genetic algorithm (GA) is employed to select the optimal parameter, so that the modified S transform has a better measure ability for S2 split. In order to evaluate the performance of the proposed method, a quantitative experiment was performed on simulated S2 signals with different split duration between A2 and P2. The effectiveness was compared with the standard S transform and another modified version of S transform.

Introduction

Cardiac auscultation is an important way for physicians to learn about patients’ health, and the second heart sound (S2) is considered as “key to the auscultation of the heart sound” [1]. In the physiological sense, S2 is generated by the asynchronous closure of aortic valve and pulmonary valve. Thereby, there is a short time delay between the aortic valve closure sound (A2) and the pulmonary valve closure sound (P2), and the time delay is called S2 split. Usually, for the healthy subjects, A2 and P2 last for less than 60ms, and the split duration varies from 30ms to 80ms [2]. However, when cardiac dysfunction occurs, this duration can be different. Therefore, the split duration provides significant diagnostic information, and the accurate measure of S2 split duration is considered as a critical topic in heart sound research.

The main difficulty of S2 split measure is the overlaps between A2 and P2 [4]. When the overlaps increase to a certain extent, (i.e., A2 and P2 come quite close), the A2 and P2 will merge into one sound, causing the failure to measure the split duration from time domain. In order to improve the measure ability of such case, variety of time frequency representation (TFR) techniques were employed.

Shanti R et al. utilized discrete wavelet transform (DWT) and continuous wavelet transform (CWT) to calculate the S2 split duration [4]; however, this paper did not provide verification of calculation accuracy. Isa Yildirim et al. used Wigner–Ville distribution (WVD) to measure S2 split [5]; however, because of the existence of cross term, WVD is not suitable for the analysis of multicomponent signal, such as S2. Bassam Al-Naami et al. employed CWT to calculate the split duration [7]; however, this paper did not concern the overlaps between A2 and P2, and it did not conduct quantitative experiments to validate the calculation accuracy. In fact, the overlaps were ignored by most of literature that studied S2 split.

Considering this, in order to improve the measure ability of S2 split, especially for the case of overlaps, this paper proposes a target function and employs a genetic algorithm (GA) to modify...
an existing TFR technique, S transform. The modified S transform has a better time concentration, resulting in the better performance of measuring the short-distance time delay (i.e., more overlaps) between A2 and P2. In order to validate the superiority of this method, a quantitative experiment was performed, and the experimental results were compared with the standard S transform and another modified version of S transform.

The rest of this paper is structured as follows. Section II explains the proposed method. Section III describes the experimental setup, and the experimental results are presented and discussed in Section IV. Finally, Section V presents the conclusions.

**Method**

**Standard S Transform**

The standard S transform is defined as Eq. 1 [8]

\[
S(t, f) = \int_{-\infty}^{\infty} h(\tau) \left| \frac{f}{\sqrt{2\pi}} \right| e^{-\frac{(\tau-\alpha)^2}{2}} e^{-i2\pi f t} d\tau
\]  

where \( h(\tau) \) denotes the input signal.

This equation can be rewritten as follows:

\[
S(t, f) = \int_{-\infty}^{\infty} h(\tau) w(\tau - t, f) e^{-i2\pi f t} d\tau
\]  

where \( w(\tau - t, f) \) denotes the Gaussian window and it is written as

\[
w(\tau - t, f) = \frac{1}{\sigma(f)\sqrt{2\pi}} e^{-\frac{(\tau - t)^2}{2\sigma(f)^2}}
\]  

\( \sigma(f) \) is the standard deviation of the Gaussian window and it is equal to \( \frac{1}{|f|} \). Note that \( \sigma(f) \) determines the width of the Gaussian window.

In practice, the standard S transform can be calculated by a fast algorithm. Assume that the Fast Fourier transform (FFT) of \( h(t) \) is \( H(f) \), then the S transform is obtained by

\[
S(t, f) = \int_{-\infty}^{\infty} H(\alpha + f) e^{-\frac{2\pi^2 a^2}{f^2}} e^{i2\pi f t} d\alpha, f \neq 0
\]  

**The modification of the standard S transform**

Even though the S transform has a lot of superiority, in some cases, it suffers from poor energy concentration in the time-frequency domain [9]. Especially for the measure of S2 split, the poor energy concentration of S transform limits its performance.

Several studies have proposed strategies to improve the energy concentration of the S transform [9], [9], [10]. The most common idea is to optimize the width of the Gaussian window by adding additional parameters to the standard deviation of the window function. In order to obtain the optimal parameters, the concentration measure (CM) function proposed in [12] was employed by most literature. This measure is defined as
where $|S_{opt}(t,f)|$ is the modulus of the normalized optimized S transform, and the normalization is performed as follows:

$$S_{opt}(t,f) = \frac{s_{opt}(t,f)}{\sqrt{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |s_{opt}(t,f)|^2 dt df}}$$ (6)

The optimal parameters were selected when $CM$ reached the maximum.

**The proposed modified S transform.** To some extent, the reported modified versions of the S transform indeed improve the energy concentration in the view of $CM$ presented in Eq. 5, however, this improvement is not suitable for the measure of S2 split. This is because that the optimal parameters that were selected based on Eq. 5 tend to bring better energy concentration under the sacrifice of time concentration, and the poor time concentration will lower the measure accuracy of S2 split and even cause the failure to distinguish A2 and P2 from each other in time-frequency domain when the overlaps are large. In this paper, the window-width optimization method is still employed to modify the S transform and the Gaussian window is optimized by [9]

$$\sigma(f) = \frac{1}{|f|^p}$$ (7)

However, the optimal parameter is selected by a novel target function which is denoted as CTC(constraint time concentration). The discrete form of the target function is defined as

$$CTC(p) = \sum_{n=0}^{N-1} \frac{1}{\sum_{m=0}^{N-1} |S^p_M(m,n)| \left(\frac{2F(m)}{fs}\right)^r}$$ (8)

where $S^p_M(m,n)$ is the modified S transform that is normalized by Eq. 6, the rows of $S^p_M(m,n)$ correspond with time and the columns denote the frequency. $F(m) = \frac{m}{2N} fs$, it is the frequency sequence; $fs$ is the sampling rate; $(\frac{2F(m)}{fs})^r$ is the punishment coefficient (or constraint coefficient), parameter $r$ controls the variation rate of the punishment coefficient as the frequency varies. For the simulated S2 signals, $r$ is set to 6, and for the real-world S2 signals, $r$ is set to 2 based on experimental results. The $p$ corresponds to the maximal CTC will be selected as the optimal parameter. Note that when $p$ is equal to 1, the modified S transform degenerates to the standard form which is shown in Eq. 1.

In order to explain $CTC$ better, the time concentration without punishment (or constrain) is defined as below:

$$TC(p) = \sum_{n=0}^{N-1} \frac{1}{\sum_{m=0}^{N-1} |S^p_M(m,n)|}$$ (9)

The idea of $CTC(p)$ can be explained as follows: for the measure of S2 split, the ideal modified S transform is required to provide energy distribution only within the duration of S2.
It means that the S transform coefficients outside the duration should come close to zero, i.e., the coefficients located in the columns outside the duration of S2 are extreme small. Therefore, for each of these columns, the reciprocal of the summation is a large value (i.e., the summation on the denominator in Eq. 9). Furthermore, the summation of all the reciprocals is also large (i.e., the outer summation in Eq. 9). From Eq. 7, it can be deduced that a larger $p$ narrows the Gaussian window, causing higher time concentration. If without constraint, the target function (i.e., Eq. 9) will select the maximum $p$ as the optimal parameter because the maximal $p$ provides the largest time concentration (see in Fig. 1(b)). However, too large $p$ will extremely weaken the frequency concentration of the modified S transform, causing the high frequencies to mistakenly contain energy, and even resulting in the fault of the time-frequency distribution (see in Fig. 2(e)). Therefore, the time concentration must be restricted (or “punished”). The punishment coefficient (i.e., $\left(\frac{2F(m)}{f_s}\right)^r$ in Eq. 8) utilized in the $CTC$ is a power function of frequency, and provides the relative small punishment in low frequency and much more punishment in high frequency, so that the target function selects the appropriate value of $p$ (see in Fig. 1(c)). The parameter $r$ is different for the real-world S2 signals and simulated S2 signals. This is because for the simulated signals, the values outside the S2 duration are zero while for the real-world signals, the values are nonzero due to the existence of noise, artifacts or murmurs. Therefore, the S transform coefficients located in these areas play the role of punishment for the real-world S2 signals. Thus, the additional punishment $\left(\frac{2F(m)}{f_s}\right)^r$ of the real-world S2 signals can be different from that of the simulated S2 signals.

Figure 1. (a), (b) and (c) are the variation of $CM$, $TC$ and $CTC$, respectively. $TC$ is the time concentration without constrain. The graph was obtained from a real-world S2 signal which was labeled as wide split from the Texas Heart Institute.
Figure 2. (a) is the S2 signal labeled as wide split. (b) is the S transform modified by CM of (a). (c) is the standard S transform of (a). (d) is the proposed S transform modified by CTC of (a). (e) is the S transform modified by TC. Notably, “NA” represents “normalized amplitude.” It can be observed the time-frequency distribution in (d) has the best time concentration.

Fig. 1 shows the variation of CM, TC and CTC calculated based on a real-world S2 signal from the Texas Heart Institute with different values of $p$, respectively. The horizontal axis represents $p$, and the vertical axis of Fig. 1(a), Fig. 1(b) and Fig. 1(c) denote the normalized CM, TC and CTC, respectively. It is observed that the optimal $p$ selected by CM is less than 1, which means the width of the Gaussian window is widened compared with the standard S transform, resulting in a lower time concentration; the optimal $p$ selected by TC is 2, and this value will cause the fault of the time-frequency distribution of the S2 signal; the optimal $p$ selected by CTC is larger than 1, producing a higher time concentration. Fig. 2 presents the standard S transform distribution and three modified S transform distribution with these three optimal values of $p$, respectively. It is observed that time distribution in Fig. 2(b) is wider than that of the standard S transform (shown in Fig. 2(c)) while the time distribution in Fig. 2(d) is tightened. Moreover, the S transform distribution in Fig. 2(e) shows an incorrect distribution of time and frequency. Therefore, the S transform modified by CTC can provide the best performance on the measure of S2 split and it has better analysis ability for the case of overlaps.

The selection of the optimal parameter based on GA. The Genetic algorithm (GA) is employed in this paper to optimize the parameter $p$. The main operations of GA usually contains encoding, selection, crossover and mutation. In addition, fitness function is also a significant item in GA. In this paper, the fitness function is the constraint time concentration (CTC) in Eq. 8.

After adding the parameter $p$, the modified S transform can be written as

$$S_M^p(t, f) = \int_{-\infty}^{\infty} h(\tau) \frac{|f|^p}{\sqrt{2\pi}} e^{-\frac{(t-\tau)^2}{2}} e^{-i2\pi f \tau} d\tau$$  (10)
As the standard S transform, the modified S transform also can be calculated by the fast algorithm as follows:

\[
S_M^P(t, f) = \int_{-\infty}^{+\infty} H(\alpha + f)e^{-\frac{2\pi^2a^2}{T^2}}e^{i2\pi at}d\alpha, f \neq 0
\] (11)

where the parameters in Eq. 10 are the same with those in Eq. 4.

Thereby, the fitness function can be simply obtained by using Eq. 8 and 11. The initialization of GA is based on the following parameters: encoding length = 10, population size = 20, crossover probability = 1.0, mutation probability = 0.01, maximum generation = 20 and \(1 \leq p < 2\).

1.1.1 The calculation of S2 split

The S2 split duration is calculated based on the modified S transform. The procedures of the calculation can be summarized as follows:

a) First, calculate the proposed S transform \(S_M^P(t, f)\) for the S2 signal.
b) Then obtain the one-dimension S-envelope using the following equation:

\[
S_{En}(t) = \int_{-\infty}^{+\infty} S_M^P(t, f)df
\] (12)
c) Normalize \(S_{En}(t)\) as follows:

\[
\overline{S_{En}}(t) = \frac{S_{En}(t)}{\max(S_{En}(t))}
\] (13)
d) Detect the local peaks of \(\overline{S_{En}}(t)\), if there are two peaks, calculate the distance of them, if the number of peaks is more than two, calculate the distance between the first and the last peak, and this distance is the S2 split duration.

Experimental Setup

In order to validate the measure accuracy of the proposed methods, a quantitative experiment was performed. Considering the simulated signal is easy to control the split duration and verify the experimental results, the simulated S2 signals proposed in [12] were adopted as the experimental material in this paper. To generate different S2 signals, the amplitudes of the simulated A2 and P2 were randomly determined from 0.8 to 1.0 and from 0.7 to 0.9, respectively. In addition, the initial phases of A2 and P2 were also randomly selected at the range of 0 to 5. The split durations were set to 30ms, 45ms and 60ms, and for each duration, ten S2 signals were generated.

Finally, the performance of the proposed method is evaluated by the average absolute errors. The experimental results are compared with the standard S transform and the method proposed in [10].

Results and Discussion

The split durations of the test S2 signals were 30ms, 45ms and 60ms, corresponding to the overlaps of 30ms, 15ms and 0ms. The experimental results are presented in Table 1. It is observed that when the split durations are 60ms and 45ms, the proposed method has the
smallest measure errors compared with the standard S transform and the modified S transform in [10]. When the split duration decreases to 30ms, the standard S transform failed to measure all of the 10 simulated S2 signals. This is because when the overlaps between A2 and P2 reach 30ms, these two components nearly merge into one sound from the time domain. In the contrast, the proposed modified S transform can still provide a satisfied measure. The measure error of the method in [10] is the smallest at the overlap of 30ms, however, the results seem a bit not reasonable, because its measure error decreases as the overlap increases, which runs counter to the common sense.

<table>
<thead>
<tr>
<th>Split duration [ms]</th>
<th>30</th>
<th>45</th>
<th>60</th>
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<tr>
<td>Error of standard S transform [ms]</td>
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<td>0.7</td>
</tr>
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<td>3.2</td>
<td>4.3</td>
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<td>Error of this method [ms]</td>
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<td>1.0</td>
<td>0</td>
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</table>

Table 1. Comparison of measure errors.

Notably, "-" represents fail to measure

Furthermore, the limit of the measure ability of the proposed method was also tested. Under the condition that is described in section III, when the real split duration is set to 25ms (i.e., the overlaps reach 35ms), the mean ± SD of the measured split duration is 23.3ms ± 4.4ms, and the mean error is 3.5ms. When the real split duration is set to 20ms (i.e., the overlaps reach 40ms), these values come to 16.2ms ± 3.5ms and 4.7ms. In fact, considering that the split duration of the real-world S2 signal varies from 30ms to 80ms [2], the measure ability of the proposed method is sufficient to provide a high accuracy detection.

Summary

This paper proposes a novel target function to modify the standard S transform, and the GA is utilized to select the optimal parameter, so that the modified S transform has the better accuracy for the measure of S2 split. In addition, by using this method, the measure limit of the overlaps are greatly improved compared with the standard S transform. To verify the method, a quantitative experiment was performed on the simulated S2 signals with different split durations. The experimental results show a reliable performance of the proposed method. This method is also suitable for the real-world S2 signal.

References


