Study on the Mathematical Relationship between the Fibonacci Sequence and Luoshu

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Abstract. In researches of basic theory of TCM, the Luoshu has direct relationship with the Jiugong bafeng theory. In this paper based on analysis of mathematical relations among the Fibonacci sequence, the Fibonacci rectangle and golden rectangle, according to geometric properties of the golden rectangle as well as numeric distribution rules implied in Luoshu, relationship between the Fibonacci sequence including its modular series by 10 and Luoshu is studied, and inherent and consistent mathematical relationship between them is found in spatial orientation, digital running sequence, numeric grouping and generating rules, etc. The study results of this paper can help to build up close mathematical association between Luoshu and the Fibonacci sequence, which can lay key theoretical foundation for the ultimate breakthrough for scientific mysteries containing in Luoshu and the Jiugong bafeng theory in TCM.

Introduction

The Fibonacci sequence is one of the most famous mathematical sequences in various fields home and abroad, and it exposes a wide range of applications in modern scientific researches [1]. The Luoshu, as one of the most famous totem for Chinese traditional cultures (as shown in Fig. 1), is also the most simple magic square of order 3 in modern mathematical system, which has been widely concerned and discussed.

![Figure 1. The Luoshu.](image)

In the research of basic theories of TCM, the Luoshu and the Jiugong bafeng theory are directly related with each other [2]. Exploration, research and analysis of the relationship between Luoshu and Fibonacci sequence based on modern mathematical principles has great
significance and values for deeper understanding of the Jiugong bafeng theory in TCM. The author has found the Luoshu and the modular Fibonacci series by 10 are closely related in mathematics based on analysis of Luoshu number distribution and also puts forward the Luo-Fibonacci diagram structure to express their profound internal relations in an issued paper [3]. And further in this paper based on analysis of mathematical relations among the Fibonacci sequence, the Fibonacci rectangle and golden rectangle, according to geometric properties of the golden rectangle as well as numeric distribution rules implied in Luoshu, relationship between the Fibonacci sequence including its modular series by 10 and Luoshu is studied, and inherent and consistent mathematical relationship between them is found in spatial orientation, digital running sequence, numeric grouping & generating rules, etc. The study results of this paper can help to build up the close mathematical association between Luoshu and the Fibonacci sequence, which can lay key theoretical foundation for the ultimate breakthrough for scientific mysteries containing in Luoshu and the Jiugong bafeng theory in TCM.

New Findings on Mathematical Relationship of the Fibonacci Sequence and TCM Theories of the Jiugong Bafeng and the Luoshu

In the research of the TCM Jiugong bafeng theory, the Luoshu numeric distribution is closely linked together with the Bafeng distribution, geographic direction & spatial position, season & solar term, etc., which can be directly used to express concepts in TCM of eight directions, eight key solar terms and the Bagua. At the same time, in Luoshu, daily running rules for Taiyi in the Jiugong is represented in sequential and increased order of natural numbers from 1 to 9 [2]. The Demonstration of Taiyi Jiugong disc for divining unearthed in Western Han Dynasty provides an important reference of materials for researches on close relationship between Luoshu and the TCM Jiugong bafeng theory [4]. As demonstrated in Fig. 2, there are straight line segments sculptured on the Taiyi jiugong disc for divining to identify the eight directions and the eight key solar terms, which is directly associated with Luoshu numeric distribution.

Figure 2. Taiyi Jiugong disc for divining.
Laws of sequential order of the Luoshu numbers from 1 to 9 as well as their position distribution according to the eight directions and eight key solar terms are of special important meaning and value for the study of the TCM Jiugong bafeng theory. Interestingly, in the Fibonacci sequence related mathematical properties by geometry and algebra, the eight directional distribution structure as well as their close contact with the number sequence order and spatial position of Luoshu number from 1 to 9 is also reflected.

Eight Directional Structure Hiding in Fibonacci rectangles and Golden Rectangles

The Fibonacci rectangle and the golden rectangle are shown in Fig. 3.

As shown in Fig. 3 a), the Fibonacci rectangle is a kind of complete covering pattern for a plane by squares, lengths of each square are all numbers from the Fibonacci sequence. In fact, the Fibonacci rectangle can also be seen as the spiral extension of rectangles with side length of successive number pairs, i.e., two adjacent Fibonacci numbers. In another words, taking the inside $2 \times 3$ small rectangle composed of two initial $1 \times 1$ squares and the square with side length 2 as the first rectangle, rotating 90 degrees in the clockwise direction to find the second $3 \times 5$ rectangle composed of the first rectangle and the square with side length 3, then rotate another 90 degrees in the same direction to find the third $5 \times 8$ rectangle composed of the second rectangle and the square with side length 5, $\ldots$, and so on. Assuming the general term of the Fibonacci sequence is $F(n)$, wherein $n$ is the index value, according to properties of the Fibonacci sequence, when $n \to \infty$, the ratio of two adjacent Fibonacci numbers $F(n)/F(n-1) = \phi$, i.e., the golden section number. Therefore in the view of the spiral pattern described above, the Fibonacci rectangle will approximate a golden rectangle gradually as index value increases and take the golden rectangle as the limit when $n \to \infty$. Actually, the golden rectangle still maintain the close contact with the Fibonacci sequence, the proportion of each side of every golden rectangles at different level of dimensions are kept algebraic combinations of Fibonacci numbers and the golden section number by the plus or minus operation, as shown in Fig. 3 b).

It is to be emphasized that there is still close relationship existed between the golden rectangle and the well-known golden logarithmic spiral. As shown in Fig. 3 there is also a similar eight directional structure like Fig. 2 formed by four separated lines across the spiral center. And in mathematics it can be proved that the intersection angle between each group of two adjacent straight lines across the spiral center is exactly 45 degrees [5]. Although there is
no similar circular structure as in the Fig. 2, considering the golden spiral closely related with the golden rectangle is an equiangular spiral and thus it essentially has great relevance with angular properties of circles in the view of rotation movements. Therefore, associating the eight directional structure in Fig. 2 with the similar spiral & eight pointed structure of the golden rectangle in Fig. 3 is reasonable in some sense, which also build up connections of the Fibonacci rectangle with Luoshu and the Jiugong bafeng theory in TCM.

**The Relationship between the Fibonacci Sequence Properties and Natural Number Sequence as well as the Digital Space Position from 1 to 9 in Luoshu.**

As the famous magic square of order 3, there are many mathematical symmetry properties in Luoshu. In addition to the sum of each horizontal and vertical and oblique direction are 15, which also satisfies sum of the triple product of all row or column is both equal to 225, i.e., the square of 15. In [6] it is found that a similar identity can be obtained if substituting the 1-9 numbers in Luoshu by the random 9 consecutive numbers in the Fibonacci sequence respectively. Namely, there is the following theorem:

Suppose \(a_1, a_2, a_3, \cdots, a_9\) be any 9 consecutive numbers in the Fibonacci sequence and substituting them sequentially to the position of number 1, 2, 3, \ldots 9 in Luoshu, the corresponding matrix can be obtained as below:

\[
\begin{pmatrix}
a_4 & a_5 & a_6 \\
a_3 & a_6 & a_7 \\
a_8 & a_1 & a_6
\end{pmatrix}
\]

Sum the triple product of the of each row or column to get two sums

\[a_4a_5a_2 + a_3a_5a_7 + a_8a_1a_6\]

and

\[a_4a_5a_8 + a_6a_5a_1 + a_2a_7a_6\]

Then we have

\[a_4a_5a_2 + a_3a_5a_7 + a_8a_1a_6 = a_4a_5a_8 + a_6a_5a_1 + a_2a_7a_6.\] (1)

This theorem associates numbers in the Fibonacci sequence with natural number order from 1 to 9 as well as the corresponding spatial positions in the Luoshu matrix which suggests that there is close mathematical link inherently between Luoshu and the Fibonacci sequence. In fact, as mentioned above, in TCM theory the daily running laws in Jiugong of the Taiyi is also following the order of natural number sequence from 1 to 9 based on their spatial distributions in Luoshu.

**Study on the Relationship between Different Arrangement Modes of Fibonacci Sequence Modulo 10 and Luoshu Number Distribution**

The Fibonacci sequence modulo 10 is a cyclic number sequence with a period of 60 below [7]:

\[
1 1 2 3 5 8 3 1 4 5 9 4 3 7 0 7 4 1 5 6 1 7 8 5 3 8 1 9 0 9 9 8 7 5 2 7 9 6 5 1 6 7 3 0 3 6 9 5 4 9 3 2 5 7 2 9 1 0
\]

If arranged in 2 by 30 mode, the 60 numbers of the Fibonacci sequence modulo 10 becomes:

\[
1 1 2 3 5 8 3 1 4 5 9 4 3 7 0 7 4 1 5 6 1 7 8 5 3 8 1 9 0 9 9 8 7 5 2 7 9 6 5 1 6 7 3 0 3 6 9 5 4 9 3 2 5 7 2 9 1 0
\]
Except the zeros in the fifteenth and thirtieth column, sum of each column are all equal to 10. The Circular arrangement of The Fibonacci sequence modulo 10 is shown in Fig. 4.

![Circular arrangement of The Fibonacci sequence modulo 10](image)

**Figure 4.** Circular arrangement of The Fibonacci sequence modulo 10.

As shown in Fig. 4, if further distributing these numbers on a circle in the direction of counter clockwise, then numbers on each diameter just corresponds to those in one of the columns, i.e., except the four zeros connecting by the black cross line in the graph, the sum of all the other number pairs on each diameter is sure to be equal to 10, which exactly coincides with the hidden rules of number pairs in the Luoshu such as 1-9, 2-8, 3-7 and 4-6 as shown in Fig. 5.

![The Luoshu along a circle](image)

**Figure 5.** The Luoshu along a circle.

Rearrange the Fibonacci sequence modulo 10 as 4 by 15 matrix as below:

\[
\begin{align*}
1 & 1  & 2 & 3 & 5 & 8 & 3 & 1 & 4 & 5 & 9 & 4 & 3 & 7 & 0 \\
7 & 7 & 4 & 1 & 5 & 6 & 1 & 7 & 8 & 5 & 3 & 8 & 1 & 9 & 0 \\
9 & 9 & 8 & 7 & 5 & 2 & 7 & 9 & 6 & 5 & 1 & 6 & 7 & 3 & 0 \\
3 & 3 & 6 & 9 & 5 & 4 & 9 & 3 & 2 & 5 & 7 & 2 & 9 & 1 & 0 \\
\end{align*}
\]

Then except columns all composed of the number 5 or 0, the other columns are completely
derived from 3971 and 2486 by necessary position shifting operations. And besides the two columns with 0 or 5, there are 15 corresponding squares can be formed as shown in Fig. 4 (For convenience of discussing below, only six of them are graphed). Therein the vertexes of each square all corresponds to a individual column in the 4 by 15 matrix. For each square whose vertexes are composed of 2, 4, 8, 6, the order of the numbers is all following the counterclockwise arranging direction of 2486 if regarding 2 as the initial number. Similarly for each square whose vertexes contains 3971, the number order is all following the clockwise arranging direction of 3971 with 3 as the initial number. They entirely coincides with the rules hidden in the Luoshu as shown in Fig. 5. Even more interestingly, for each pair of squares in Fig. 4 with the same color, the numbers corresponding to their vertexes coincide with a relative location of the eight Luoshu numbers except the center number 5 in topology.

In [8] numbers in the Luoshu are also divided into two groups corresponding to different squares in Fig. 5, and associates them with the arithmetical compliment sequence 3971, 2486 and 5 respectively obtained by the integer power of 3, 2 and 5 modulo 10. Wherein for numbers 5, no matter how much the power is, the obtained arithmetical compliment is all kept the invariant number of 5, therefore which is located at the center of the Luoshu. So, the numbers 2, 3 and 5 can be seen as the core gene numbers in some sense based on the generating idea of Luoshu to obtain the arithmetical compliment sequences by the integer power of 2, 3 and 5 modulo 10. In fact, the number of 2, 3 and 5 are just three consecutive numbers in the Fibonacci sequence which can form a basic recursive & circulatory body to generate the Fibonacci sequence. In this way, the Fibonacci sequence can undoubtedly establish certain mathematical relationship with the generation mode of Luoshu.

Conclusions and Future Researches

In this study, the related geometric and algebraic properties of the Fibonacci sequence as well as its modular sequence by 10 can have profound mathematical relationship with the numeric order and spatial distribution of the Luoshu numbers. In [3] the author have also established association between the Luoshu structure and the key sequence derived by the Fibonacci sequence modulo 10 and produced a graph structure named the Luoshu-Fibonacci diagram, in which the Luoshu number distribution is also directly included. In [9], the author has also introduced a novel graph named the diagram of Nine-mansions and Xian-Tu, which associates the 3 by 3 grid structure of the Luoshu with the golden spiral, the golden rectangle as well as the Xiantu. The new findings in this paper together with the related research results of the two literature by the author have established solid connections between the Luoshu and the representative golden phenomena of the Fibonacci sequence, golden spirals and golden rectangles, which can lay mathematical foundation for the scientific decipher of the ancient Luoshu and TCM Jiugong bafeng theory, in the future researches the author can integrate all of these findings above together to conduct in-depth study and discussion. Considering the Fibonacci sequence is widely existed and applied in various disciplines of modern sciences especially in biology disciplines, the study of this paper has great theoretical value and practical significance for exploring the modern scientific rationality of the TCM basic theory.

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